# Journal

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# The Franklin Institute

AUGUST, 1953

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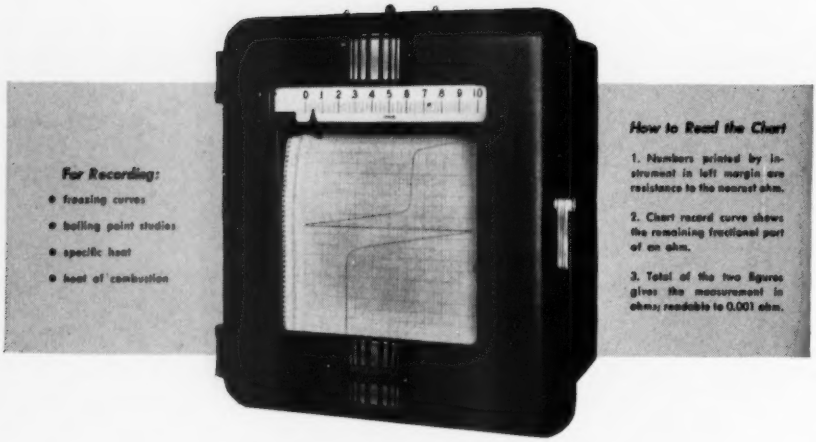
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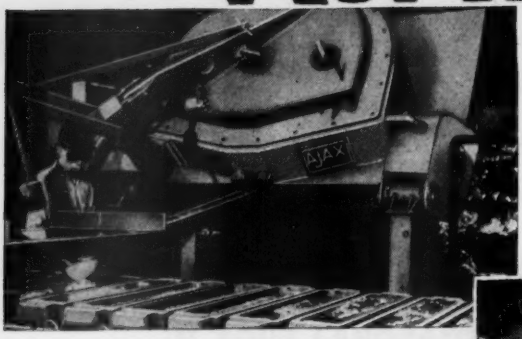


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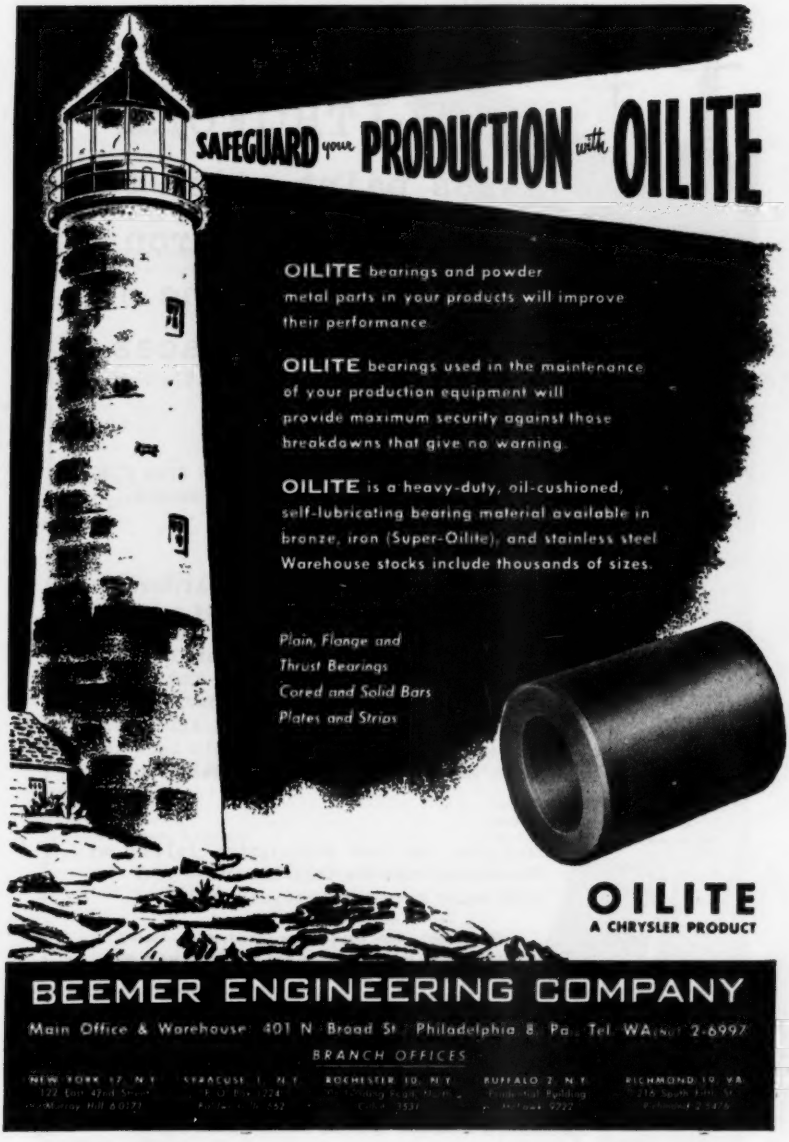
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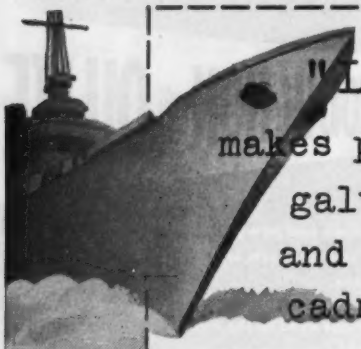

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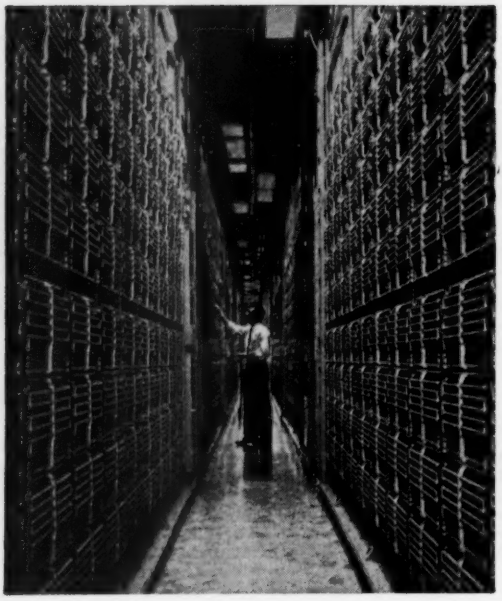

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The film takes the place of impregnated paper formerly used to separate the metal foil electrodes. It is tougher, stands more voltage and needs no impregnation. The new capacitors require no protective housing and are *much* smaller and less costly.

Here is a new example of the way America's technology advances through sharing knowledge. Just as Bell Telephone Laboratories makes many of its discoveries — the Transistor, for example — available to other companies, so does it adapt inventiveness of others to help your telephone service.

*Unrolled view (reduced size) of capacitor unit wound with "Mylar." The transparent film is only 0.0005" thick yet stands handling without breaking.*

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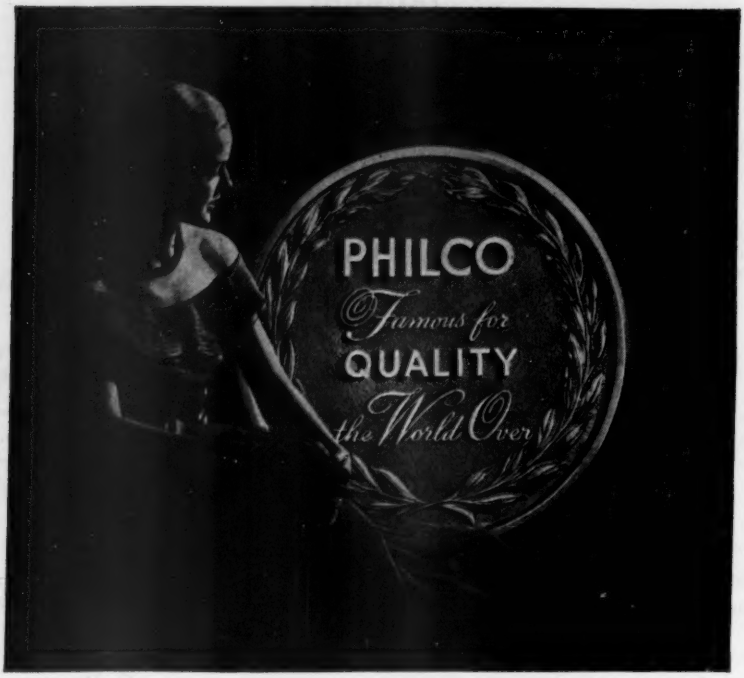


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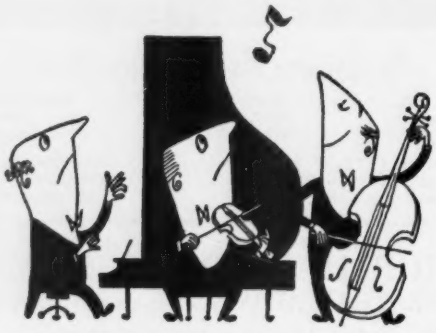

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**Devoted to Science and the Mechanic Arts**

**Vol. 256** **AUGUST, 1953** **No. 2**


## VERMILYE MEDAL PRESENTATION*

**The Franklin Institute, Wednesday, May 20, 1953**

At the presentation of the Vermilye Medal, made at the Stated Meeting of The Franklin Institute, held May 20, 1953, in the Lecture Hall, Mr. S. Wyman Rolph, President of the Institute, opened the proceedings, as follows:

MR. ROLPH: "Since the Edward G. Budd Lectures have first been given, it has been our privilege to hear from Dr. George L. Kelley, of England, who is with us this evening; Mr. William L. Batt, who spoke on the subject, 'Vision Knows No Boundaries'; and Mr. Harold Stassen, whose subject was 'One Freedom, Indivisible.'

"This evening, it happens that the Edward G. Budd Lecture will be given by the recipient of the Vermilye Medal. So, in order to expedite matters, I am going to present to you Mr. Ralph Kelly, Chairman of the Vermilye Medal Committee of the Institute's Board of Managers, who will tell us about the recipient of the Medal and at the same time you will have heard more about our speaker of the evening. Mr. Ralph Kelly."

MR. KELLY: "It is my privilege as Chairman of the Vermilye Medal Committee to tell you briefly about the Medal and to present the recipient to you.

"William M. Vermilye founded the Vermilye Medal as an award of The Franklin Institute in recognition of outstanding contributions in the field of industrial management. Practically all of the other awards of The Franklin Institute are for scientific inventions or for the development of the theories such as those expounded by Einstein or Milliken. William Vermilye believed that The Franklin Institute should, at frequent intervals, recognize great industrialists for their work in the field of management. William M. Vermilye was a staunch American of

* Proceedings of the Vermilye Medal Award, presented to Benjamin F. Fairless, May 20, 1953, at The Franklin Institute.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Dutch descent. He was a member of the Class of 1901 at Massachusetts Institute of Technology. He acquired a great deal of experience in the textile business, being one of the pioneers—in those days, quite rare—in successful labor-management relations. At the end of his career, he served as a Vice-President of the National City Bank and he was one of those delightful people whom it is always a pleasure to meet. He was a great friend of The Franklin Institute. The Vermilye Medal Committee of 1953 is composed of Edward G. Budd, Jr., President of The Budd Company, James Creese, President of our great technical

Benjamin F. Fairless (left) receives the Vermilye Medal from S. Wyman Rolph.

college, The Drexel Institute of Technology—both of whom are here tonight—Charles Penrose, and myself as Chairman. The previous Medalists comprise a small but very distinguished group. The first Medal was awarded to Louis Brown in 1939, who was then President of Johns-Manville. He was followed by William Knudsen, head of war production at a critical period of World War II and also the great production genius of General Motors. Then came Walter Gifford, the famous President of A. T. & T. and, most recently, Ambassador to Great Britain. The next one was Martin Clement, who is here tonight, the former President of the Pennsylvania Railroad and a great Philadelphian. The most recent Medalist was Howard Pew, President and

Chairman of Sun Oil Co., who was such a factor in the growth and development of the oil industry in our country. Mrs. Pew is here tonight, but Mr. Pew could not be with us. You notice that these Medalists have all had different characteristics, but they were great administrators.

"Ladies and Gentlemen, I could talk at great length about the Medalist of tonight, Benjamin F. Fairless. I could say that he was born in Pigeon Run, Ohio, that he was brought up as a poor boy, and I could give you other details about his history which practically all of you know. It is a long record and a most creditable one. Probably his greatest achievement in management is in unifying and coordinating the bulky, over-grown and very independent components of the United States Steel Corporation into a unified company with reasonably straight organization lines and with each department working with the others. It was one of the great organization jobs of all time and it had a marked bearing on the splendid job the United States Steel Corporation did in World War II for America. Wars could not be won without steel and we would not have won World War II without the steel provided by the United States Steel Corporation. One of Mr. Fairless' great policy decisions—and to us in Philadelphia, his greatest—was to locate one of their major mills in the Philadelphia area, in order to participate to a larger extent in our important Eastern market. It was fitting that the plant be named the Fairless Plant, after the individual who had the responsibility for the move. It was an important and a wise decision. Mr. Fairless deserves the thanks of all Philadelphians, not only for spear-heading the industrialization of the growth of our Northeast area, but also for the increased growth that will result to our port. But to me and to all patriotic Americans, Benjamin Fairless will go down in history as the leader of a group consisting of Clarence Randall and other steel company presidents which preserved our way of life, when an attempt was made to seize the steel industry. The attempt would have been successful, had it not been for the courageous stand of this group. We who prize the American way of life owe a great debt to Benjamin Fairless and to those other leaders of the steel industry. I will now read the citation for this Medal:

> "In recognition of his outstanding contribution to the field of Industrial Management by further developing and unifying the great number of diverse units and industries within the structure of the United States Steel Corporation to form an efficient and progressive organization; and for the fairness and understanding with which he has conducted difficult and confusing labor and industrial negotiations, earning him the respect of the industrial world. Overcoming early hardships to become the head of one of the largest corporations in the world, he is an outstanding example of the American way of life and the free enterprise system.

"Mr. President, as Chairman of the Vermilye Medal Committee, I present to you Benjamin F. Fairless, whom our Committee recommends for the Vermilye Medal for 1953."

MR. ROLPH: "Mr. Fairless, by virtue of the power vested in me as President of The Franklin Institute, I present to you the Vermilye Medal for 1953 and the accompanying certificate, with my very warm congratulations, sir. Before I turn the meeting over to you, I have failed to mention that Mrs. Budd is with us this evening, as are Edward G. Budd, Jr., and his wife. We are very happy to have them with us and I am sure that great honor is being done to them and to Mr. Budd, whose memory we are honoring in having you here to give the Edward G. Budd Lecture. Thank you, Mr. Fairless."

After thanking Mr. Rolph, Mr. Fairless delivered his address, "Mid-Century Miracle." This talk (see this JOURNAL, p. 109), served the dual purpose of acknowledging the receipt of the Vermilye Medal and of honoring the memory of Edward G. Budd.

# MID-CENTURY MIRACLE *

BY

BENJAMIN F. FAIRLESS [1]

Your kindness to me this evening has been so overwhelming that I hardly know what to say. To receive the Vermilye Medal of The Franklin Institute is one of the highest and most welcome honors that has come to me in my professional life; and there is simply no way to thank you properly for all of the generous things that have been said about me here.

When I received Dr. Allen's letter informing me of this Award, I was naturally delighted; but I was also perplexed—as you can well imagine—by certain doubts and misgivings which have persisted ever since.

Now I would be the last man in the world, of course, to question the unfaltering wisdom, and the infallible judgment, of your Medal Committee; but when I think of all the business and industrial leaders who have contributed so much to the welfare and the progress of our nation during the past year, I cannot help wondering whether it is possible for any group of mere mortals to determine truly which one of these men is most deserving of an honor such as this.

In fact, it has even occurred to me that the conscientious members of your Committee, themselves, may share my doubts concerning the decision they have made; and if so, I want them to know, at least, that while they could undoubtedly have named others more worthy of this Medal than Ben Fairless, they could not possibly have found anyone, anywhere, who would appreciate it more!

So it is in the consoling light of that reassuring thought, Ladies and Gentlemen, that I accept here tonight—with pleasure, with pride, with heartfelt gratitude, and with a deep sense of humility—the distinguished Award which you have bestowed upon me.

But I am doubly honored on this occasion by the invitation to deliver the Edward G. Budd Lecture; for I can think of no more welcome assignment than to join you here in paying tribute to the life, the work and the memory of this man who contributed so importantly to the growth and development of American industry and enterprise.

The brilliant record of his achievements has been a source of endless inspiration to us who survive him; but that, of course, is a story which has

* Presented under the Edward G. Budd Lecture Foundation of The Franklin Institute; given in the Lecture Hall at the Stated Meeting held May 20, 1953. The Franklin Institute's Vermilye Medal for leadership in the field of industrial management was awarded to Mr. Fairless on this occasion.

[1] Chairman of the Board of Directors of the United States Steel Corporation.

already been told so fully and so capably by Dr. G. L. Kelley, who spoke from this platform three years ago, that no one, I think, could hope to enlarge upon it successfully.

So I have been asked to devote my remarks tonight to a discussion of the responsibilities of present-day industrial management towards the community at large; and as that is a favorite topic of mine, I am most happy to comply.

Now Dr. Kelley has told us that Edward G. Budd made it his practice to ask the impossible of one of his people each day; and that quite often they did it.

Well, that was not surprising to Mr. Budd, I am sure, because he was quite a hand at doing the impossible himself; but it is even less surprising to us today. We live in an age of miracles, and they have become so commonplace in our industrial world that if any day passes without producing some new and wonderful accomplishment, we are inclined to worry about it, and we wonder if maybe we aren't beginning to slip a little.

At this midway point in the Twentieth Century, we have the miracle of radio and television and atomic energy. Then there is the even greater miracle of mass production, to which we undoubtedly owe our very survival as a free and independent nation. But tonight, I want to talk with you about another mid-century miracle which may seem to you to be a very minor one; but which—to my way of thinking—may prove to be the most significant of them all.

And that is the miraculous reversal which has occurred in the public attitude towards business management in the past few years.

Twenty years ago—at the depth of the depression—the American businessman was an outcast. So far as public opinion was concerned, he found himself at the very bottom of the barrel; and while it is true that he shared this unhappy position with a number of unfortunate bankers and a few unreconstructed Republicans, he was generally regarded, nevertheless, as the lowest form of economic life.

It was said that since he ran his business for private gain, he could never be trusted to run it in the public interest. It was argued that anything that was good for him was automatically bad for the Country. And it was generally agreed that he must be shackled with endless regulations and controls.

In fact, had it not been for the part he played in winning the war, when American industry was called upon to outproduce all of the enemy nations put together—and did so—I honestly believe that our whole system of free, individual, competitive enterprise would have been overwhelmed, here in this Country, by the same kind of state socialism that has swept so many lands abroad.

But that did not happen. The miracle occurred!

Today, industrial management is winning the renewed confidence of the American people. The right to manage its own affairs has been largely

restored to it; and some of its leading figures have been called upon to help establish—not a businessman's administration, but a business-like administration—in Washington. But this new prestige—enjoyable as it is—has brought heavy obligations with it; and personally, I know of no leader of business and industry who is not acutely conscious of them.

The will of the people, I think, has been made crystal clear. Just as they want their government to be run in a business-like way, so they expect American business to be run in the long-range interests of the public. And the major responsibility of present-day industrial management—as I see it—is to meet this expectation fully; for should we fail to do so, I greatly fear that many of the basic individual liberties that we have cherished in this nation for the past 175 years will be crushed forever by the growing authority of an all-powerful state.

But any discussion of the grave responsibility which rests upon the shoulders of American businessmen today must begin, it seems to me, with a recognition and an understanding of the great fundamental change which has taken place—even during my lifetime—in the character of industry itself, and in the nature of modern industrial management.

Back in the last century, even the largest of our industrial enterprises were often owned by a single individual or family; or by a little handful of men acting as partners in the venture. And—as is always the case, I suppose, in any century, and in any broad occupational group—these men were of all kinds and dispositions, good, bad, and middling.

The only kind we ever seem to hear about any more, of course, is the bad kind—the men who were selfish and ruthless, and who were interested only in making what is known today, I believe, as a "fast buck." We seldom hear of the other kind—the men who were driven by a passionate desire to build . . . the men, in short, like Edward Budd, who found their greatest satisfaction just in doing the impossible.

But no matter what kind of a man the owner was—nor what his motives might be—his authority was absolute in those days. He ran the business himself; and therefore could run it to suit himself. As long as he served his customers to their satisfaction, and obeyed the relatively simple laws that were then on the books, he was responsible only to his conscience and could do just about as he pleased.

Every dollar that he paid to labor came out of his own pocket; and if his workers became unruly, he could shut down his plants, if he wished, and try to starve them into submission. If he could contrive to extract an extra dollar from his customers, it was just so much velvet for him. And if he could drive a competitor out of business—by fair means or foul—he gained a new measure of security for himself and his family.

He played for high stakes, and the stakes were his own. If he lost, he lost everything—his business, his savings, his home and his shirt. If he won, he won big; and sometimes became so enormously wealthy that the

splendor in which he lived was in almost-shocking contrast to the comparative poverty in which his workers existed. And thus the successful owner came to be known as a "robber baron" and later as a "malefactor of great wealth."

Yes, this was the conventional picture of the man who brought business leadership into popular disrepute, and whose sins have been visited upon industrial management right down to the present generation.

But let's face it, Ladies and Gentlemen: He did build America! He made it the most productive nation on the face of this earth. And, in spite of himself, he did more to give the American people their present high standard of living than all the reformers who ever lived!

So if we are to understand fully the great change that has occurred in industrial management during the past half century, I think we must first get rid of a few of the misconceptions that still persist about these so-called "robber barons."

Let us remember, for example, that they did not create the industrial conditions of their day. They inherited them from their predecessors; for industry—in one form or another—has existed on this earth since the earliest days of recorded history, and for many centuries the labor force consisted chiefly of slaves.

Let us also remember that nineteenth-century working conditions in America—poor as they may have been by modern standards of comparison—were nevertheless the finest in the world at that time. That is why immigrants from almost every other country on earth flocked to our shores by the millions to seek their fortune here, and to enjoy a standard of living and a degree of security that they could never have hoped to gain through a lifetime of work in the industries of their native lands.

But granting all that, let us agree that wages, compared to those of today, were pitifully low even in America . . . that in order to keep body and soul together, men sometimes had to labor almost beyond the limits of their physical endurance, and that little children had to work long hours in the mills. Let us also agree that these conditions bred disease . . . that the safety of workers in dangerous occupations was not properly protected, and that there was a great deal of inhumanity to man, generally speaking. And then let us ask ourselves: "Why?"

Why did these conditions exist? Was it—as Karl Marx declared—because greedy owners stole what he called the "surplus value" produced by the workers?

No. That clearly is not the answer; for if all the wealth of the owners had been distributed equally among their workers, the resulting improvement in the living standards of the men would have been negligible at best, and even this improvement would not have lasted long.

The real trouble was that the workers—for all their long hours of labor—could produce so little with the tools they had at their disposal in those

days, that the total value of the things they made was barely sufficient to provide them with the minimum necessities of life.

Why, just think what would happen if modern America—with its 160 million people—had only the tools which existed back in the days when Benjamin Franklin coaxed that historic spark of lightning down his kite string. Wages would fall to a few cents a day. Hours of work would lengthen unbearably. And it is doubtful whether all of the men, women and children in the land—toiling the clock around—could produce enough food, shelter and clothing to sustain the life of our entire population. Many would surely die; millions would suffer from malnutrition and exposure; disease would spread everywhere, and all the socialists, all the reformers, all the labor unions and all the agencies of government could never change it!

No, Ladies and Gentlemen, the truth is that the high wages, the short hours, and the enlightened working conditions that industrial labor enjoys in this country today, have not been brought about by the purely-humanitarian impulses of anybody—discouraging as that thought may be to all of us.

Our workers have gained more wealth, simply because they produce more wealth; and they are able to produce more, simply because modern tools and methods of production have multiplied their strength enormously. Today, each of these workers is the boss of an invisible crew of mechanical men whose energies he commands at the flick of a switch, and whose productive effort enriches his life.

How did he get these tools? Who created and developed them? Who placed them at his disposal?

Well, I'm very much afraid that it all began with what we know as the "industrial revolution"; and that the job was done primarily by those same tycoons who have since been so universally and indiscriminately condemned. The personal fortunes which they amassed were a major source of the venture capital that was risked to develop and build these tools. It was their wealth that perfected the steam engine, that produced the internal combustion engine and the oil to run it, that made mass production possible and that built huge hydraulic presses, integrated steel mills, and the aluminum industry.

Let us say that their motives were sometimes selfish. Let us say that some of them hungered for fortune and power. But let us admit, honestly, that this was their doing. And, ironically, it also proved to be their undoing; for in the end, the tools and machines which they built became so big, so complicated and so expensive that even the richest of men no longer possessed enough wealth to acquire them alone.

A hundred years ago, the average worker had about $500 worth of tools at his disposal. Today, however, he must use as much as $100,000 worth of tools and machines in some of our largest industries. And a single company must often employ many thousand such workers.

That is why it is no longer possible for one individual or for one small group of individuals to own any large enterprise in America. And that is also why the dynasty of the tycoons has ended forever. It fell—not because of the greed or the ruthlessness of which we so often accuse these men, but because they literally worked themselves out of a job! They served their purpose and they passed—leaving in their place, the modern corporation.

Today, most of our big companies are owned by hundreds of thousands of people; and several large corporations have more owners than employes. But since it is not humanly possible for all these owners to run the business themselves, they must hire professional managers to do the job for them.

And that is the great, fundamental change that has taken place in the character of industry and management during the present century. Big business is no longer run directly by the men who own it; and it is no longer owned primarily by the men who run it.

The industrial manager of today is a "hired hand" who no longer has a purely proprietary interest in the enterprise. Let me hasten to add, however, that he does, of course, have a direct and compelling personal stake in it, because his entire career, his reputation and his very livelihood depend upon his ability to manage the company successfully—to keep it sound and prosperous and to provide for its necessary growth and development. And the better he does this job, the greater his reward is likely to be.

But the profits of the business, as such, no longer belong to him; the payroll no longer comes out of his own pocket; and the failure of a dozen unfortunate competitors can no longer assure his future welfare and security. So he has nothing whatever to gain by starving his fellow workers, gypping his customers, or indulging in the practice of monopoly.

It is true, of course, that he is usually a minor stockholder in the company; but in relation to the vast sums which have been invested in it by all the other owners, his own holdings are necessarily so small as to be negligible in comparison. His dividends, therefore, constitute an equally negligible part of the total profit of the business; and he has no hope on earth of ever building the kind of personal fortune that sprang from industry in days gone by.

In fact, many of the top executives in some of our largest corporations have spent a lifetime in the field of industrial management without ever having been able to accumulate as much as a million dollars. And I know that to be a fact because I happen to be one of them, myself.

So like any other employe, the professional manager can succeed only by serving his bosses loyally and to their satisfaction. But he has a great many such bosses, and they often appear to be sharply at odds with each other. In my case, for example, I figure that I have about 700,000 of them. First there are the 285,000 shareowners who hire—and can fire—me. Then there are more than a hundred thousand customers who have to be kept

happy if the corporation is to stay in business. And beyond that, my 300,000 fellow workers are not always too easy to please!

As you see, therefore, the life of the industrial manager is not exactly simple. He stands in the great, unhappy middle. His stockholders want larger dividends; his workers want more pay, and his customers want low prices, high quality and immediate delivery! And his job—Heaven help him—is to please them all!

Fortunately, however, the conflict among these three groups of bosses is far more apparent than real; for their long range interests are actually identical.

Since the worker's earning power depends on the tools that he uses, it is to his advantage that profits be large enough to attract the capital necessary to provide him constantly with better ones. But since the tools will not yield a maximum profit unless they are skillfully, efficiently and steadily employed, it is to the owner's advantage that wages be high enough to attract the finest type of workers to their plants. And since the customer will buy only where he can get the best products at the lowest prices, it is vital to both the worker and the owner that prices be kept at the lowest possible level.

So the responsibility of modern, professional management—as I see it—is to operate the company to the joint advantage of all three of these groups. It must serve their common interests at all times. But in doing this, it necessarily serves the public interest; for the customers, employes and owners of American business are the American public, and their interest is the public interest.

And that is a point which I want to emphasize especially. I know of no way by which any large enterprise in this country can survive today unless it is managed in the long-range public interest . . . unless it is ready and able at all times to meet the essential needs of its customers, in peace or in war; and unless it can provide for its future growth, in the service of generations yet to come.

Ladies and Gentlemen, as I look at American industry today, I am satisfied that management has done a good job in meeting these difficult responsibilities. Clearly it has been fair to the owners because they continue to supply it with the most productive and efficient facilities that exist in the world today. Clearly it has been fair to the workers because they enjoy the highest wages and the richest standard of living that the world has ever known. And clearly it has been fair to the customers because the price of our products—measured in terms of the hours of work necessary to acquire them—is the lowest on this earth.

Yes, modern professional management has served the public interest. It has provided the jobs, the tools and the goods which our people have needed to sustain their economic welfare and their national security; and I should be inclined to be pretty smug and complacent about our industrial future today were it not for one thing:

And that is this unending strife between management and labor.

Here—to my way of thinking—is the most difficult problem and the most pressing responsibility that the managers of American industry face in this day and generation. We pay, as I have said, the highest wages in the world. We spend hundreds of millions of dollars each year to protect the health and safety of our workers and to help provide for their future security. We spend additional millions on the betterment of human relations within our plants—to make work more pleasant, more rewarding and more satisfying to the men who perform it. But still we have failed to secure industrial peace.

We are confronted by the discouraging fact that there were more strikes in this Country last year than ever before in its history; and it seems to me that the leaders of management, on their part, would do well to study those strikes, to determine their causes, to search out the errors that were made, and to avoid their repetition in the future. And with a little soul-searching of the same kind on the part of labor's leadership, we might—between us—manage to avoid this useless, senseless conflict. For there is one thing we do know: That strikes are never in the public interest; and nobody ever wins them.

The workers lose, the owners lose and the customers lose—and so do the workers, owners and customers in many other companies which are not directly involved in the dispute at all. In short, everybody loses; and certainly all of us would be better off if strikes did not occur.

Why, then, do they continue; and what can industrial leadership offer by way of preventing them?

Well, my friends, I wish I knew; for if I could answer those questions, I would really deserve a medal. But since I can't, I should like in the few minutes remaining to me here this evening, to give you—for what they may be worth—one or two personal observations and opinions based on my own experiences over the years.

A wise philosopher once said: "More is needed to deal with a single individual in our times, than a whole people in the past"—and while he made that observation about four centuries ago, its truth is obviously timeless.

And so today, it seems to me that our greatest opportunity, on the management side of the picture, lies in the field of human relations. We must be quick to sense human aspirations and to find the means by which those aspirations can best be realized in this age of mass production. We must also strive to maintain and improve constantly our industrial communications so that both management and men may come to understand each other's problems better.

But above all, we must recognize frankly, I think, that no matter how earnestly the leaders of industry may seek to do so, they will never succeed in gaining industrial peace until the leadership of labor displays an equal

desire to attain it; for industrial peace—if we are to achieve it at all—must be based firmly on mutual confidence, respect and understanding.

Now in saying that, I realize that it is always easier to see the mote in the other fellow's eye; but it seems to me that the time has come when responsible labor leadership ought to rid itself of two old-fashioned and outworn ideas which seem to lie at the root of most of our present difficulties, and which should have disappeared along with the "robber barons."

One of these is the wholly-unrealistic notion that labor and management are natural and eternal enemies; and that I, for example, am in some way a foe, or antagonist, of the men who work in our mills. Actually, of course, we are fellow employes of the same company, performing different assignments in our mutual interest, as the nature of our business requires.

The other is this disastrous idea that in order to bargain collectively, we must sit on opposite sides of a table and try to take something away from each other. Our real task, of course, is to join together in producing more wealth, not only for ourselves, but for our whole national economy.

We shall never produce more wealth, however, by fighting over the profits of the owners; nor will our workers ever gain wealth and security for themselves by trying to whittle those profits away; for if we persist in doing that, then one of two things must happen: Either we must go on raising wages on the one hand, and then having to boost prices, on the other—a course which results only in the phony illusion of prosperity—or we shall end up with inadequate profits, poorer tools, less production, lower wages, diminished security, and finally, widespread unemployment.

So it is time we recognized, I think, one simple economic fact that is true under the system of production in any country of the world: And that is that industry's capacity to pay higher wages is not governed primarily by the company's profits; but that it depends entirely upon the value of what the worker produces with the tools that he uses.

But the tragic truth in America today is that industry's capacity to pay could be greatly increased if it were not for strikes and slow-downs and feather-bedding work rules which restrict the output of men and machines and limit their earnings accordingly.

If all these restrictions upon the productive power of labor could be wiped out, once and for all, I firmly believe that many of the workers in many of our industries—using exactly the same tools and machines that are now at their disposal, and without lengthening their hours of work in any way—could substantially increase their present earning power. And they could do this, moreover, without necessitating any increase in prices that would add to their cost of living.

Here, then, is a field which affords almost unlimited opportunity for enlightened cooperation between the leaders of labor and management. I am not suggesting, of course, that labor should assume any of the responsibilities and functions of management, or that management should step in

and try to run the union; but I am suggesting that if we can ever get away from strikes, disputes, and blunderbuss bargaining, we have a magnificent opportunity to start working together in our own—and in the public—interest.

Ladies and Gentlemen, I believe that we can, that we shall, and that we must find the way to industrial peace. I see hope in the fact that—just as a great and welcome change has taken place in the character of industrial management during the past fifty years—so a similar change may now be taking place in the character of labor's leadership.

I believe that the most successful and most powerful labor unions of the future will be those which never call a strike at all, and never have to do so. They will fulfill their labor contracts faithfully. They will strive—not to destroy—but to promote mutual confidence and understanding between management and worker. And they will insist, above all else, that the only limitations which shall ever be placed upon the productive power and the earning power of any of their members shall be the individual skill, the ability and the capacity of the man himself.

With that kind of union leadership, and with responsible, present-day management, there is no conceivable limit to America's progress. There is no standard of living so high that our people cannot attain it.

# PLASTIC STRESS-STRAIN RELATIONS UNDER RADIAL AND NON-RADIAL COMBINED STRESS LOADING *

BY

JOSEPH MARIN [1] AND JOHN A. SAUER [2]

SUMMARY

An experimental investigation, using tubes subjected to internal pressure and axial loads, has been made of plastic stress-strain relations for an aluminum alloy. Results are presented for various types of loading, including tests where constant stress ratios were maintained and other tests involving variable stress ratios. In the "radial" loading cases, tests were run and data obtained at five different values of the principal stresses, ranging from pure circumferential tension to pure axial tension. A number of non-radial loading paths were studied and the data plotted in the form of true plastic stress versus true plastic strain curves. It was observed that the principal strains depended greatly on the path of loading used to reach a given principal stress state. It was also observed that onset of yielding was approximately in accord with a shear energy criterion and onset of fracture with a maximum stress criterion. Interpretation of the plastic stress-strain data for all tests has been attempted by use of a $J_2$ type of incremental plasticity theory.

## INTRODUCTION

The behavior of materials in the plastic range has been extensively studied for materials or test specimens subjected to simple tension. Knowledge of such behavior is, however, insufficient to predict performance under more complicated states of stress, such as are encountered in most machine or structural members. It is essential, therefore, to investigate the behavior of materials under various combinations of combined stress. One of the most convenient methods of doing this is to subject a specimen, in the form of a thin-walled cylinder, to a combination of axial tension and internal pressure. This is the method used in the present investigation. Most previous investigators, however, have confined their attention to "radial" loading, that is, loading in which the biaxial combined stress ratio is maintained constant; but in this study, stress-strain data are obtained for variable combined stress ratios as well.

The specific material used for both the constant and the variable stress-ratio tests was an alloy of aluminum designated as Alcoa 14S–T6. Information concerning its composition and properties, as well as details

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of test procedure, testing apparatus, and strain-measuring instruments, are given elsewhere (1).[3] In the present paper only some of the more interesting results of our investigation will be presented.

### CONSTANT STRESS-RATIO TESTS

In these tests under so-called "radial" loading conditions, the ratio of the principal stresses was maintained constant throughout the duration of the test by maintaining a prescribed ratio of internal pressure to axial tensile load. Stress-strain data were taken for five different ratios of biaxial principal stresses (designated either by $\sigma_2/\sigma_1$ or $\sigma_T/\sigma_a$, with the numerator standing for the tangential or circumferential component

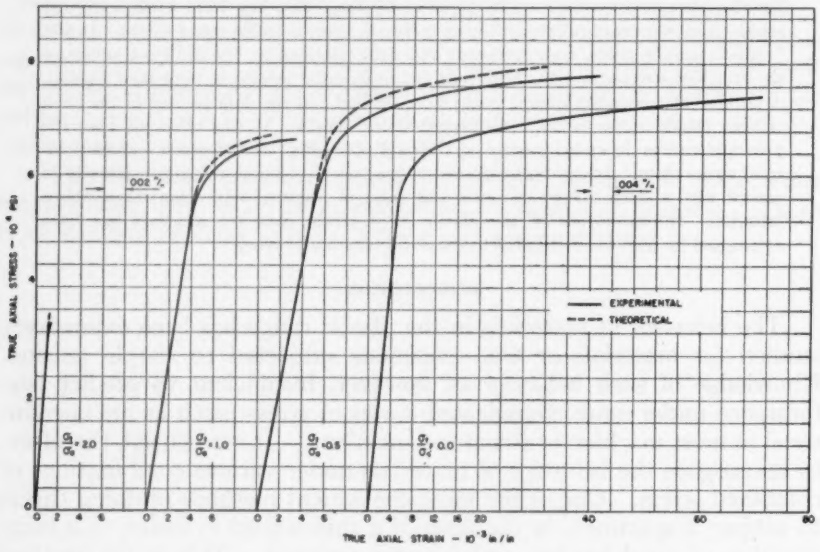


FIG. 1. Comparison of true axial stress-strain relations with plasticity theory for constant stress ratio tests.

of stress and the denominator for the axial component of stress). The stress values investigated were $\sigma_T/\sigma_a = 0$, corresponding to pure axial tension; $\sigma_T/\sigma_a = \frac{1}{2}$, $\sigma_T/\sigma_a = 1$, corresponding to uniform tension in both directions; $\sigma_T/\sigma_a = 2$, and $\sigma_T/\sigma_a = \infty$, corresponding to pure tension in the tangential direction. This latter stress state was produced by means of a special fixture, described in detail elsewhere (1), which eliminated the axial stress caused by the internal pressure.

In the stress-strain graphs to follow, the stress values plotted are true stresses based on the value of the applied loads and the instantaneous values of the specimen dimensions. Likewise, the strains are true axial or true tangential strains as the case may be. Figure 1 shows

[3] The boldface numbers in parentheses refer to the references appended to this paper.

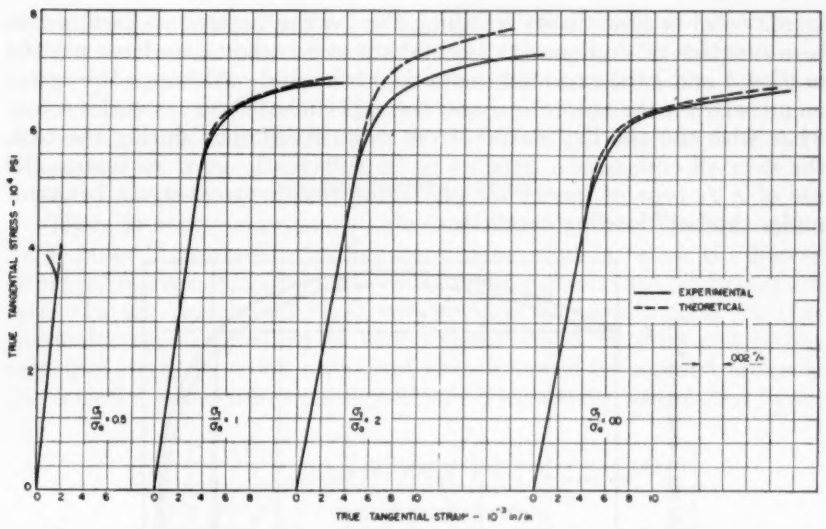


FIG. 2. Comparison of true tangential stress-strain relations with plasticity theory for constant stress ratio tests.

the experimental data in the form of a series of four axial stress-strain curves, one for each of the four radial loading paths investigated. Figure 2 presents analogous experimental plastic stress-strain relations for the tangential direction.

An attempt has also been made to interpret the experimental data in terms of standard theories of plasticity. Using a simple type of

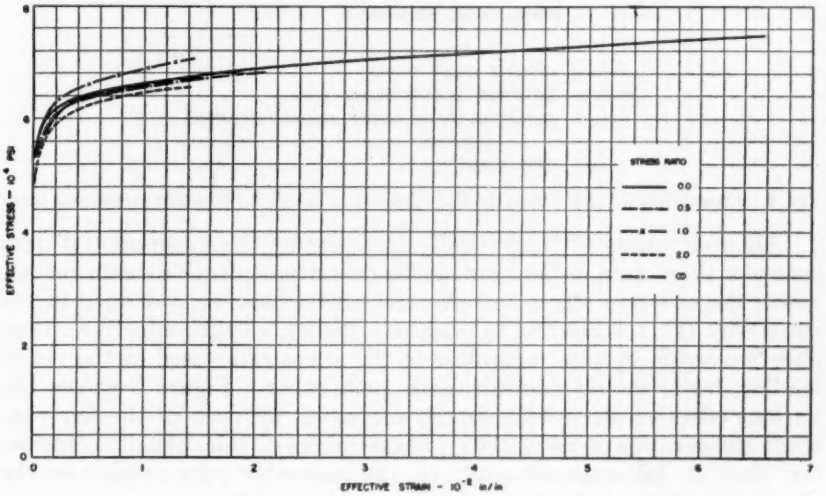


FIG. 3. Effective stress-strain relations for constant stress-ratio tests.

incremental or flow theory in which the loading function is assumed to be a function of $J_2$ alone (2) (second stress invariant), we have plotted in Figs. 1 and 2 the expected theoretical behavior. Although the agreement between the theoretical and the experimental curves varies somewhat with the specific biaxial stress ratio maintained during the test, the over-all comparison appears to be sufficiently good to justify the use of a $J_2$ type of theory for predicting plastic stress-strain behavior under "radial" loading conditions.

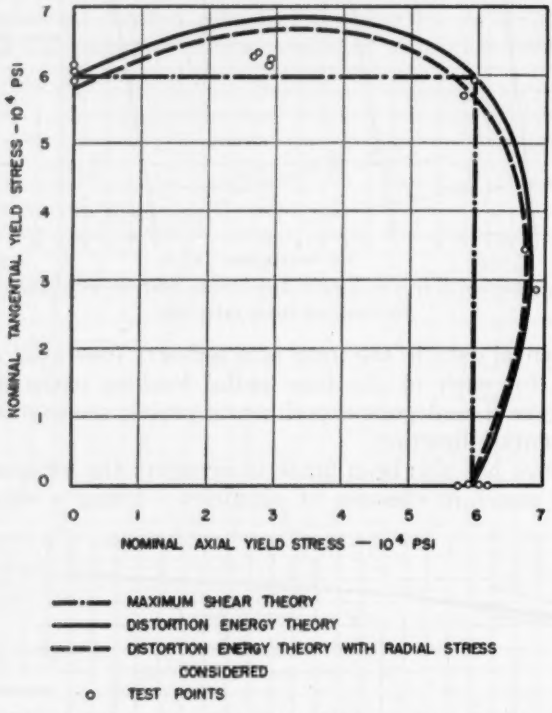


FIG. 4. Comparison of yield strengths with theories of failure for constant stress-ratio tests.

Another means of theoretically interpreting the experimental data is to plot those data in the form of effective stress-effective strain curves, where the effective stress and effective strain have been defined in the usual way (1). According to plasticity theory, all such effective stress-effective strain curves, regardless of the stress ratio used in the radial loading tests, should coincide with each other. Figure 3 shows the various effective stress-effective strain curves for each of the five constant stress-ratios investigated. These curves do not actually coincide, but they do fall approximately on the curve for pure tension as the simple incremental plasticity theory would require.

During each of the constant-ratio combined stress tests, yield

strengths were determined by an equivalent offset method (3). These yield strengths are plotted in Fig. 4 on a conventional principal-stress type representation along with the expected yield behavior based on various theories of failure. The experimental data are in approximate agreement with the predictions of the so-called Distortion Energy Theory.

### VARIABLE STRESS RATIO TESTS

There is no conventional method of obtaining plastic stress-strain data under "non-radial" loading conditions, that is, when the biaxial stress ratios are not maintained constant throughout the duration of the test. In the present study, it was decided to procure data for that particular type of loading in which stress would be produced in one principal stress direction only, and then followed by applied stress in the opposite principal stress direction. These tests were called "two-

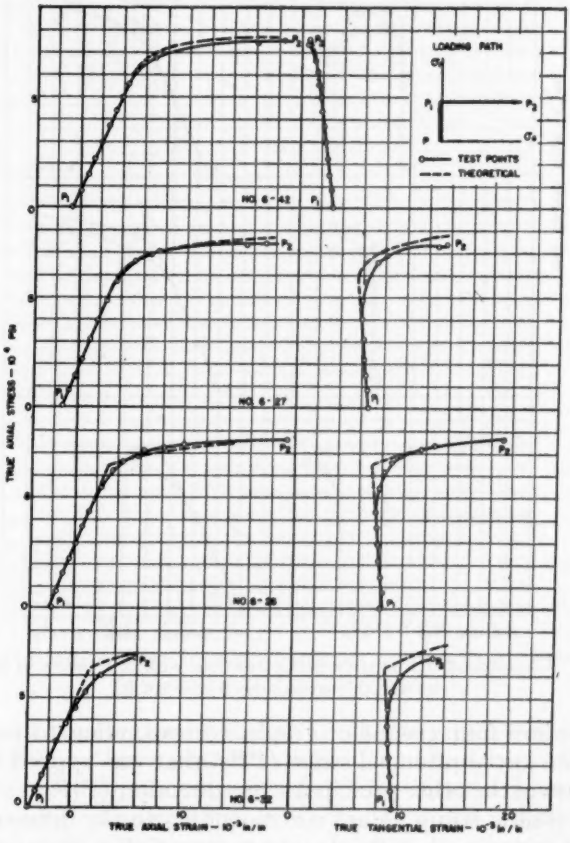


FIG. 5. Comparison of true stress-strain relations with flow theory of plasticity for variable stress-ratio tests—Set A.

step" tests, and in one set (Set A), the initial loading was applied in the tangential direction while in a second set (Set B), the initial loading was applied in the axial direction.

The data, plotted in the form of true stress-true strain curves, are shown for Set A and Set B in Figs. 5 and 6, respectively. On each

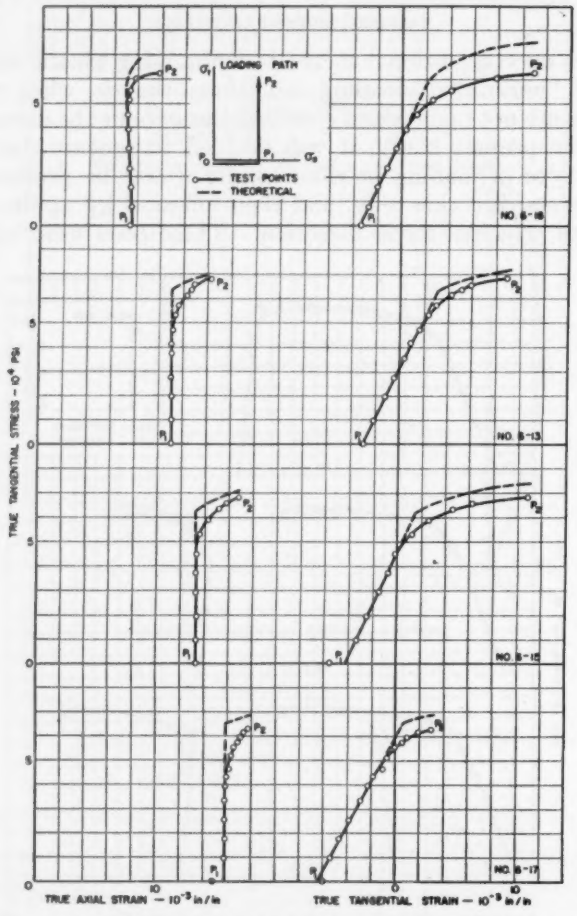


FIG. 6. Comparison of true stress-strain relations with flow theory of plasticity for variable stress-ratio tests—Set B.

figure there are four groups of graphs corresponding to four different values of the initial principal stress ($PP_1$) that was applied before loading to failure in the other principal stress direction ($P_1P_2$). The specific stress and plastic strain values corresponding to the different values of $PP_1$ will be given more fully elsewhere (1). Suffice it here to show an attempted theoretical interpretation of the data.

It has been pointed out by Drucker (*2*) and others that, as the loading path becomes more complicated, more complicated incremental stress-strain theories than that associated with the loading function $F(J) = J_2$ will undoubtedly be needed. Nevertheless, because of the simpler form and ready adaptability to design problems of the $J_2$ type of theory, it seems worthwhile to determine how well such a theory

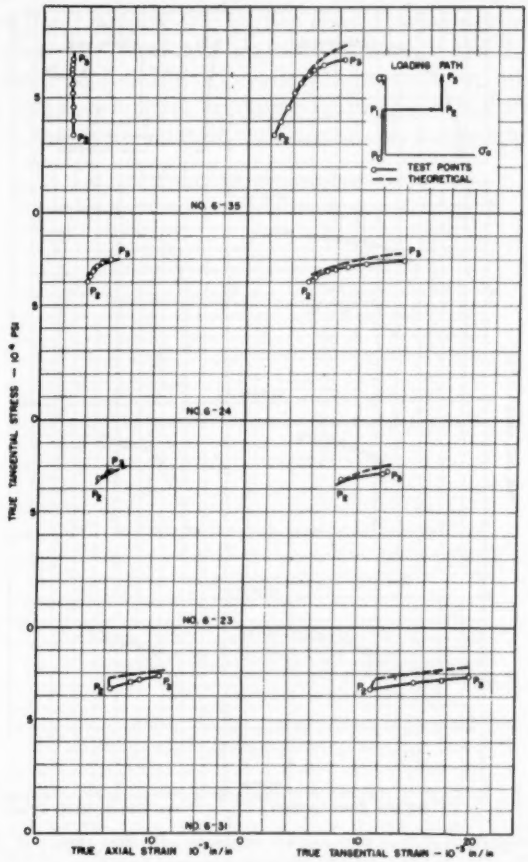


FIG. 7. Comparison of true stress-strain relations with flow theory of plasticity for variable stress-ratio tests—Set C.

could represent our experimental data for the variable stress-ratio tests. For this purpose we have plotted in Figs. 5 and 6, along with the full-line curves representing the experimental data, dotted-line curves representing the expected theoretical stress-strain behavior based on a $J_2$ type of incremental plasticity theory. Although the over-all agreement is by no means precise, a comparison of the curves seems to indicate

that, to a first approximation at least, a $J_2$ type of theory does provide a fairly accurate interpretation of the test data.

A second group of variable biaxial stress-ratio tests were also run. These tests, called the "three-step" tests, consisted of loading in one principal stress direction only ($P_0P_1$), followed by loading in the second principal stress direction ($P_1P_2$), and finally loading again in the first

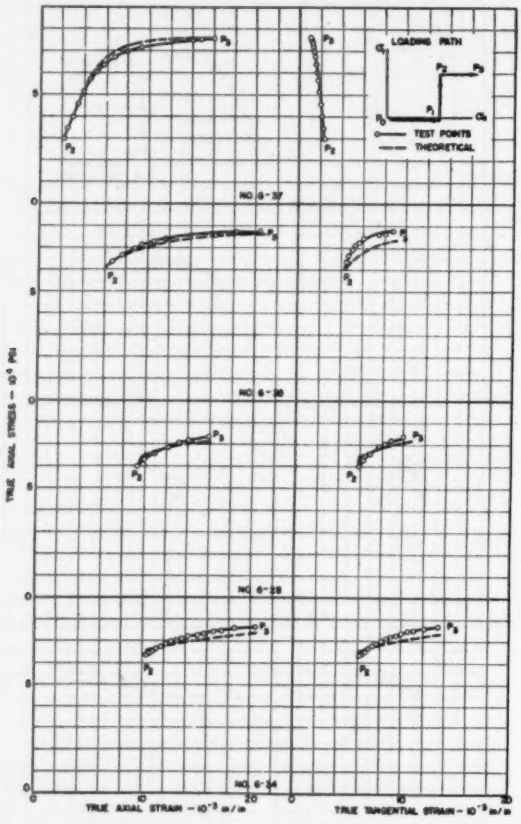


FIG. 8. Comparison of true stress-strain relations with flow theory of plasticity for variable stress-ratio tests—Set D.

principal stress direction ($P_2P_3$) until fracture took place. As in the "two-step" tests, two different sets of tests were run, Set C with initial loading in the tangential direction, and Set D with initial loading in the axial direction.

The experimental data are given in Figs. 7 and 8 where the stress-strain values obtained experimentally for the $P_2P_3$ portion of the loading process are indicated by the open circles and their general trend by the

full lines. Once again in each of the figures there are four groups of stress-strain diagrams corresponding to four different values of the initially applied loading ($P_0P_1$). Also shown on the figures, by the dotted lines, are the expected theoretical plastic stress-strain relations as determined from a $J_2$ type of incremental plasticity theory.

The agreement between the experimental and theoretical curves is perhaps somewhat closer than for the "two-step" tests and, hence, it again appears that, despite the rather complicated non-radial loading path of the "three-step" tests, a plasticity theory of the $J_2$ type is sufficient to provide a reasonably accurate interpretation of the data.

One other point which is apparent from the experimental data is that the strain values associated with any given stress state in the $\sigma_1\sigma_2$

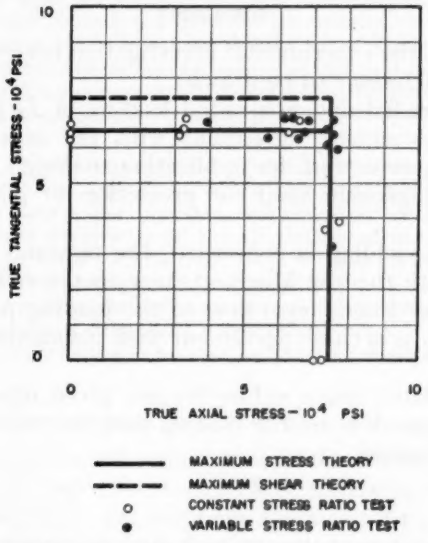


FIG. 9. Comparison of true fracture stresses with theories of failure for constant and variable stress-ratio tests.

plane depend markedly on the loading path by means of which that particular stress state was reached. For both the "two-step" and "three-step" tests the same $\sigma_1\sigma_2$ stress state could be reached by a variety of loading paths and a comparison of either the axial or tangential strains for different loading paths (but the same final state) immediately shows very large differences—amounting in some cases to several hundred per cent. It is clear, therefore, that no theory of plasticity, such as the so-called "slip theory" of Batdorf and Budiansky (4), which assumes or requires that the strains for each final loading state be dependent only on that state and not on the loading path used, can be successfully applied to the interpretation or prediction of data of the

type presented in this investigation. On the other hand, plasticity theories of the flow or incremental type do not require that plastic strains be dependent upon the final state of stress and, hence, can satisfactorily account for the observed strain dependence upon loading path.

A final graph, Fig. 9, presents the fracture stress data for all tests, including those carried out at constant stress ratio (open circles) as well as those carried out under variable stress ratios (full circles). Both sets of experimental points appear to be in close agreement with a maximum stress theory of failure. Thus the path of loading apparently has little effect on the true fracture strength even though, as mentioned previously, it has a very great effect on the plastic strain values corresponding to any given non-fracture stress state.

## CONCLUSION

The results of the experimental investigation herein reported permit the following conclusions to be drawn:

1. For both radial and non-radial loading, a $J_2$ type of plasticity theory is in approximate agreement with the experimental results. Hence it would appear that, for application to design, a $J_2$ type theory could be advantageously used for prediction of plastic stress-strain behavior.

2. Insofar as yielding is concerned, the material seems to behave in accordance with the von Mises or shear energy criterion. For fracture, on the other hand, regardless of the loading path followed, the material behavior is in closer agreement with the maximum stress theory of failure.

3. The associated strain values for any given principal stress state are markedly dependent on the loading path by which that particular state has been reached.

*Acknowledgment*

The authors acknowledge with thanks the assistance rendered by L. W. Hu and J. F. Hamburg in carrying on this study and wish also to express their appreciation to the Office of Army Ordnance for their financial sponsorship.

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# APPLICATIONS OF MICROWAVES IN SCIENTIFIC RESEARCH

BY

HANS HERBERT KLINGER[1]

SUMMARY

Microwaves are very short radio waves covering the range roughly between $\lambda = 10$ cm. and 1 mm. wave lengths. This spectrum of the electromagnetic waves, which bridges light and radio waves, holds a great deal of scientific interest. The physicist is especially anxious to study the absorption spectra of gases, and the magnetic and dielectric properties of liquids and solids in the microwave region; astrophysicists are interested in measuring the microwave radiation coming from the sun, the stars and clouds of gas in space; the biologist wants to gain a better understanding of living matter by studying the behavior of biological tissues and bacteria in the radiation field of centimeter and millimeter wave lengths; and nuclear physicists harness microwaves to accelerate electrons and protons for bombarding nuclei and to study nuclear reactions.

Before World War II comparatively little was known about the physical and scientific applications of microwaves. Although these waves have been produced to a limited extent for many years by means of spark gap generators, Barkhausen-Kurz tubes and magnetrons, the greatest advances in the field of microwave technique took place during the recent war as a result of the concentrated efforts of teams of scientists engaged in the development of microwave radar. Vastly improved new methods for the transmission, generation, reception and measurement of microwaves have provided the physicist with important new research tools. It is the aim of this paper to review the most important progress made in this new science of microwave physics.

## PRINCIPLES OF MICROWAVE TECHNIQUE

Microwave experimental techniques differ fundamentally from those used at longer wave lengths. These differences arise from the fact that in the microwave region of the spectrum the wave lengths are of the same order of magnitude as the dimensions of circuit components.

### *Wave Guides*

The most fundamental circuit component used in microwave technique is the wave guide (1)[2], a hollow pipe of conducting material. Microwaves can travel inside such a pipe if the wave length is comparable with the dimensions of the cross section of the pipe. However, electromagnetic waves inside a wave guide do not travel with their free-space velocity. The wave velocity of an electromagnetic wave inside a wave guide is greater than the wave velocity in free-space and the wave length inside a wave guide is greater than the free-space wave length.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

The various transmission modes, which are possible in a wave guide, are classified into two main types, *TM* (or *E*) modes and *TE* (or *H*) modes, depending on whether there is a transverse magnetic (or longitudinal electric) or a transverse electric (or longitudinal magnetic) field. Each wave-guide mode is characterized by a critical wave length $\lambda_c$, above which the mode can not be propagated. The value of this critical wave length depends on the shape and the dimensions of the

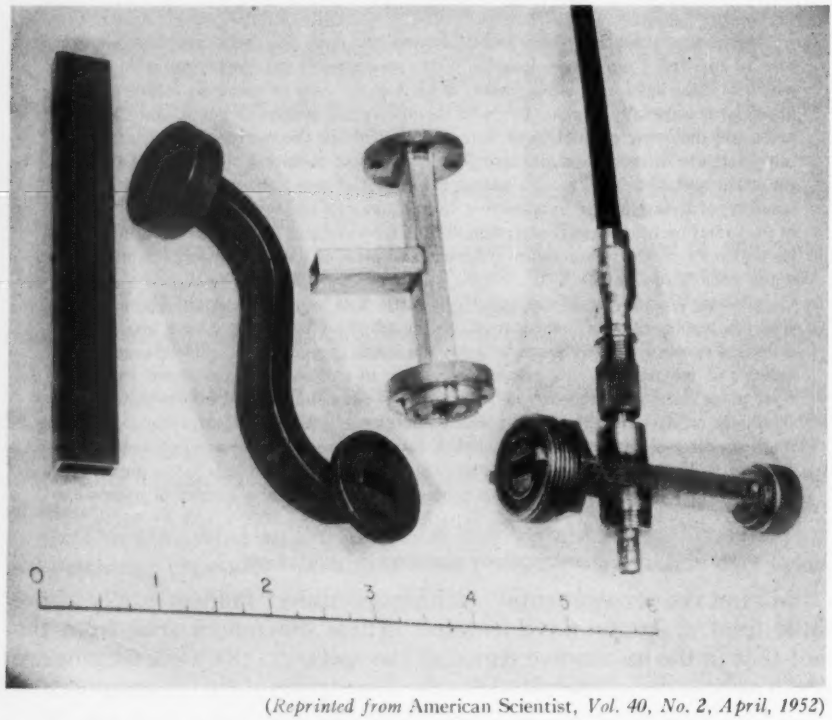


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FIG. 1. Wave guides and a T-junction for transmitting microwaves of about $\lambda = 1$ cm. wave length. A crystal detector mount is shown on the right. Scale is in inches.

guide, and on the order of the mode. For example, the critical wave length of the fundamental mode in a rectangular guide, the $TE_{01}$ (or $H_{01}$) mode is $\lambda_c = 2b$, where $b$ is the wave-guide width, while for the $TE_{01}$ mode in a circular guide $\lambda_c = 1.64a$, where $a$ is the tube radius.

Normally the simplest transmission modes, having the longest critical wave lengths, are used in practice. The most convenient mode used in microwave technique is the $TE_{01}$ (or $H_{01}$) mode in a rectangular pipe. This wave-guide mode possesses a linearly polarized electric field, which can be generated simply. Once such a mode is introduced into the pipe, the traveling electromagnetic field may be guided around

bends or the wave may be split into different arms of a wave-guide $T$-junction or a hybrid. Some important wave-guide components, used for transmitting microwaves of about $\lambda = 1$ cm. wave length are shown in Fig. 1.

*Cavity Resonators*

Because the wave length of microwaves is comparable with the dimensions of circuit components, ordinary inductance-capacity circuits can no longer be used because of excessive radiation losses. In microwave technique these circuits are replaced by cavity resonators, which are the electromagnetic analog of acoustical resonators. Such resonators consist of pieces of wave guide short-circuited at both ends, as shown in Fig. 2. The most convenient modes used in microwave

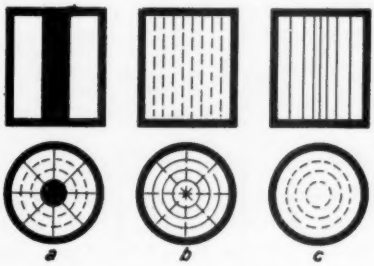



FIG. 2. Cavity resonator fields. (*a*) Coaxial half-wave resonator; (*b*) $H_{011}$ resonator; (*c*) $E_{010}$ resonator.

—— Electric field
- - - - - Magnetic field

measurements with a cylindrical cavity resonator are the $TE$ (or $H$) modes, in which the electric field $E$ has only components perpendicular to the axis of the cylinder. The magnetic field $H$, being perpendicular to $E$, has components concentric with the axis. The natural modes of oscillation of such a resonator occur when a half-wave guide length is contained an integral number of times in the distance $l$ between the ends of the cavity, that is, when

$$\lambda_g/2 = l/p, \text{ or } \lambda_g = 2l/p$$

where $\lambda_g$ is the guide wave length, and $p$ is an integer.

Cavity resonators have very high $Q$-factors. That means these resonators oscillate freely a great many times before the amplitude of the oscillation is reduced to a small value. To obtain a large $Q$ (a high selectivity), the ratio of volume to area of the cavity must be made as large as possible. If, in addition, the skin losses of the surface of the cavity are small, which can be accomplished by silver plating the inner walls of the cavity, $Q$ factors of the order of 50,000 can be obtained with cavity resonators in the microwave region.

*Microwave Oscillators*

Microwaves are usually produced by means of klystrons and magnetrons. In these electron tubes the transit time of the electrons is comparable with the period of microwave oscillations.

One of the most important microwave sources, widely used in scientific research is the reflex klystron (2). In the reflex klystron (Fig. 3) the electron beam from a cathode passes through the gap in the center of a resonant cavity and is velocity-modulated by the oscillating cavity field. The electrons emerge into the drift space, travel towards

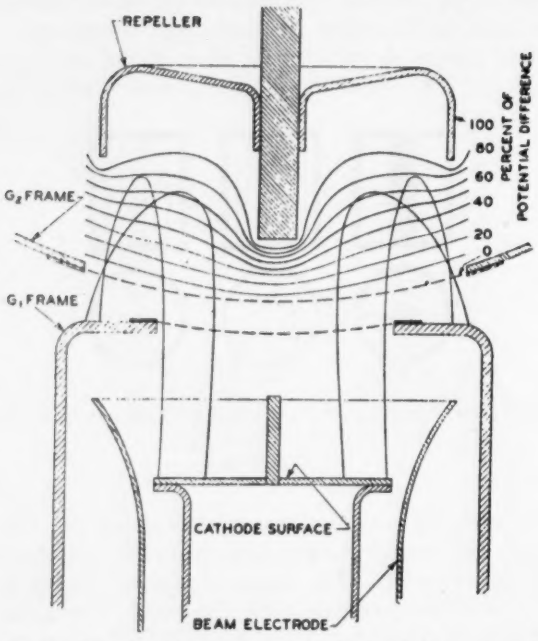
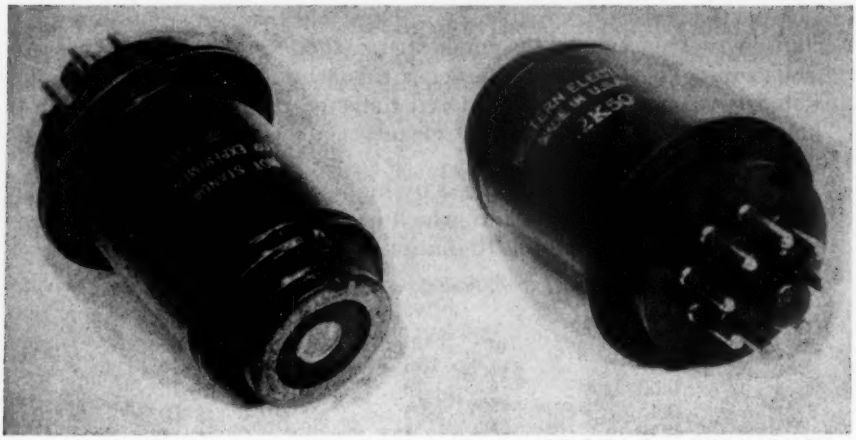



FIG. 3. Section detail of a reflex klystron tube showing typical electrostatic field plot and calculated electron paths. The cavity resonator is not shown in this figure.

the reflector, where they are repelled by a negative field and return toward the gap. During the transit time through the drift space the electrons bunch due to the velocity modulation, and arrive at the gap of the cavity resonator at the right instant to stimulate the cavity oscillations. Thus energy is transferred from the electron beam to electromagnetic oscillation in the cavity. The frequency of the reflex klystron is varied mechanically by changing the shape of the resonator by pressure, or by screwing in a tuning plunger. Alternatively, the frequency may be swept over a small range by varying the voltage applied to the reflector. Two reflex klystrons which generate microwaves of about $\lambda = 1.23$ cm. wave length are shown in Fig. 4. The shortest

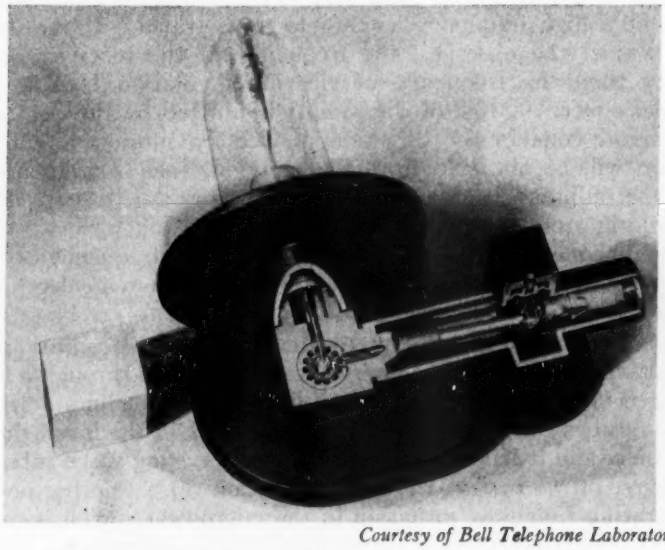



*Courtesy of Bell Telephone Laboratories, Inc.*

FIG. 4. Reflex tubes 2K50 designed for $\lambda = 1.23$ cm. wave length (Western Electric).

wave length generated by a reflex klystron is of about $\lambda = 4$ mm. (3). The dimensions of such a tube are very small and the output power is of the order of a few $\mu W$.

Very short and much more powerful microwaves are generated by means of cavity magnetrons (4). Figure 5 shows such a microwave oscillator. It consists of a copper block, in which are cut small slots and holes each forming a cavity resonator. Under the influence of an axial magnetic field and a high anode voltage, electrons emitted by an



*Courtesy of Bell Telephone Laboratories, Inc.*

FIG. 5. Cavity magnetron 725A for $\lambda = 3.25$ cm. wave length.

oxide-coated cathode capable of a large electron emission describe orbits in the space between anode and cathode and excite the cavity resonators. The microwave power is taken by a matched wave guide excited by means of a glass-enclosed tungsten probe connected with a "halo" loop over a resonator by means of a short length of a coaxial line.

Usually magnetrons are pulsed by high anode voltages. The table below shows some of the results which have been achieved with pulsed magnetrons at the Columbia Radiation Laboratory.

| Wave Length, cm. | Peak Power, KW. | Efficiency, per cent |
|---|---|---|
| 10 | 3000 | 60–80 |
| 1 | 100 | 25–30 |
| 0.63 | 30 | 15–20 |
| 0.45 | 10 | 5–10 |
| 0.30 | 2 | 1– 5 |

At the present time a wave length of $\lambda = 1.1$ mm. is the shortest undamped microwave obtained by magnetron harmonics (5). Although the peak power at these wave lengths is only a few $\mu W$, it is adequate for a variety of physical measurements.

Still shorter wave lengths may possibly be produced by methods based on the radiation of accelerated electrons. One of these methods is the "relativistic Doppler" method (6). Here a stream of high-velocity electrons is shot between the poles of a series of magnets which alternately produce magnetic fields in opposite directions. These fields deflect the electrons back and forth normal to the direction of motion at a high frequency as the electrons travel. If the electrons move toward the observer, the frequency of the received radiation is larger then the frequency of electron oscillation because of the "Doppler effect." Most of the radiation emitted by the electrons lies in a narrow cone in the direction of electron motion. While some radiation will be obtained from the individual electrons of a smooth beam, the radiation is greater if the electrons travel through the magnetic field in groups or bunches.

A similar method proposed for converting the frequency of a centimeter wave to a higher frequency is by reflection from an electron cloud traveling with relativistic velocity (7). It seems also to be possible to generate very short waves by means of Čerenkov radiation. This radiation takes place when a charged particle moves through a medium at a speed faster than the velocity of light in that medium. In traveling-wave tubes (8) the electrons travel faster than the increasing wave at the helix and it has been suggested that traveling-wave tubes might be a source of Microwave-Čerenkov radiation (9). Another possibility in generating Čerenkov radiation in the microwave region is to send a velocity-modulated beam of electrons through a microwave delay lens

where the wave velocity of the microwaves decreases. Although these methods of generating short electric waves are of great interest, they have not been investigated in sufficient detail.

*Detection of Microwaves*

Detection of microwaves is usually accomplished by crystal rectifiers (**10**). These contain a piece of silicon and a very fine wire called a "cat whisker" which makes contact with the silicon (Fig. 6). The detector is usually inserted into a wave-guide crystal holder, as shown in Fig. 1. Microwaves reaching the crystal detector produce a voltage in the detector which is proportional to their intensity. Silicon rectifiers perform reasonably well even at the shortest microwaves.

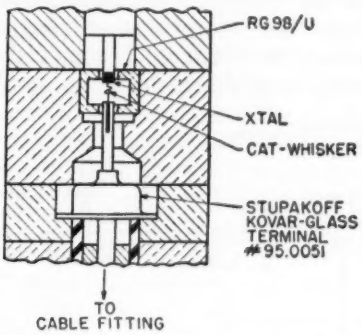



FIG. 6. Section detail of a microwave crystal detector.

## MICROWAVE SPECTROSCOPY

One of the most important applications of microwaves in physical research is microwave spectroscopy (**11**). From quantum mechanics it is known that the various possible electron configurations of individual atoms and molecules are each characterized by a discrete energy. Atoms or molecules may absorb or emit energy in discrete quantums given by the quantum relation

$$E_2 - E_1 = h \cdot \nu$$

where $\nu$ is the frequency of the quantum, $h$ is Planck's constant, and $E_2$ and $E_1$ are the total energies of the atom or molecule before and after emission or absorption of the quantum. In the microwave region the quantum energy of a frequency $\nu = 30.000$ Mc/s. ($\lambda = 1$ cm.) is $h\nu = 1.24 \times 10^{-4}\ eV$. Although this quantum energy is very small in comparison to light waves or Roentgen-rays, it is of the same order of magnitude as the lowest rotational energy states in a molecule. If microwaves having a wave length of this magnitude are transmitted through a gas, energy is absorbed from the radiation at frequencies for which the above quantum relation holds.

Before proceeding to an examination of the spectra observed, let us indicate how these spectra are measured. Figure 7 shows the arrangement of a simple microwave spectrometer. The waves are generated in a reflex klystron, traveling through a wave guide-absorption-cell containing the gas to be measured, and received by a crystal detector. The gas is as a rule at a very low pressure—approximately $10^{-5}$ atmospheres—and is isolated from the atmosphere by means of mica windows at the ends of the absorption cell. The frequency of the microwaves traversing the gas can be varied by altering the voltage applied to the reflex klystron. If the gas absorbs some particular frequency, the power at this frequency transmitted through the absorption cell is diminished as indicated by a reduced voltage produced by the crystal

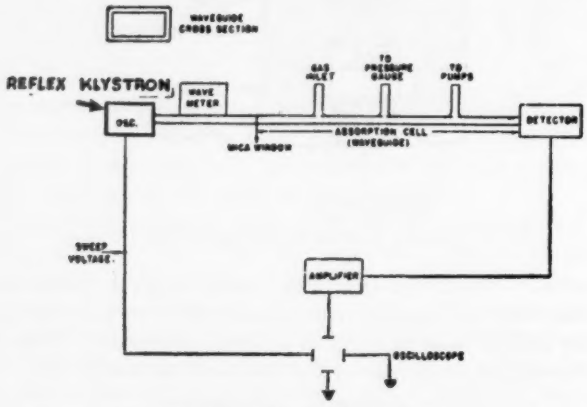



FIG. 7. Schematic plan of a simple microwave spectrometer.

detector. The reduction of voltage is observed on a cathode-ray oscilloscope. The voltage which is applied to the reflex klystron, in order to change the frequency, is also applied to the horizontal plates of the oscilloscope. Hence, the horizontal axis corresponds to the frequency variation. The output voltage of the detector is applied to the vertical plates of the oscilloscope so that the vertical deflection indicates the power reaching the detector. Any decrease of the power corresponds to an absorption. The oscilloscope then shows directly a plot of the gas spectrum in the frequency region over which the reflex klystron is being swept. The absorption frequencies can be measured to an accuracy of one part in $10^4$ by using cavity-wavemeters and to an accuracy as great as about one part in $10^7$ by comparing the microwave frequency with highly accurate quartz-crystal oscillators.

The absorption spectra of gases observed in the microwave region are mainly due to the transitions between the neighboring rotational energy states of the molecules. For a simple diatomic molecule the rotational spectrum consists of a series of frequencies which are har-

monics of the lowest frequency $h/4\pi^2 I$, where $h$ is Planck's constant and $I$ is the moment of inertia of the molecule. For many molecules, the lowest frequency falls in the microwave region. In contrast to the infrared region, where the rotational spectrum is superposed on vibrational spectra and is seen as a more or less continuous band, in the microwave region usually only one individual line is observed. In addition, the resolution of a microwave spectroscope is higher than that of an infrared spectroscope by a factor of about $10^3$ and this makes it possible to investigate in detail very small regions of the spectrum.

Microwave spectroscopy has so far been used primarily for the determination of fundamental physical and chemical properties of matter. Since the frequencies at which absorption takes place can be measured to one part in $10^7$, the accuracy of determination of the moment of inertia is limited only by the small uncertainty in the value of Planck's constant, $h$. The moment of inertia depends upon the masses of the atoms of the molecule and upon the distances between them. If the masses are known, a very accurate measurement of internuclear or interatomic distances can be obtained.

There are other important features of microwave spectroscopy. One of these is the "Stark effect," which takes place when rotating molecules are placed in an electric field. The field exerts a torque on the molecule because of its interaction with the electric dipole moment of the molecule. This torque twists the molecule and affects to some extent its rotational motion with the result that the line is split into several components. The magnitude of this splitting depends on the strength of the electric field and on the size of the electric dipole moment of the molecule. Hence a measurement of the amount of splitting can be used to determine the molecular dipole moment of the molecule. A similar splitting, called the "Zeeman effect," occurs if a molecule is placed in a magnetic field. The magnitude of Zeeman-splitting makes it possible to determine the magnetic moment associated with the molecule as a result of its rotation.

Perhaps the most interesting effects in microwave spectroscopy are associated with hyperfine splitting of the absorption lines as a result of various nuclear effects. Study of hyperfine splitting yields information about the structure of the nuclei present in the molecule.

There are other important applications of microwave spectroscopy not only in pure physics but also in technology. Spectroscopy at still longer wave lengths exists, such as molecular beam spectroscopy, nuclear resonance and others, which have much in common with microwave spectroscopy but which are beyond the scope of this paper.

## DIELECTRIC DISPERSION IN POLAR LIQUIDS

Since Drude's classical researches many investigations have been carried out on the absorption of radio waves in solids and liquids. Later these investigations were greatly stimulated by Debye's analysis

of the dielectric relaxation of polar molecules. Such molecules give rise to "dipole polarization" by orientation of the molecules in an applied electric field. The molecules of gases, being free to rotate, follow the field more or less instantaneously, and the oscillation frequencies occur in the infrared, extending into the microwave region. In solids, however, the rotational freedom is greatly restricted by the interaction of adjacent molecules, and the orientation phenomenon degenerates into aperiodic oscillation under high friction, occurring at the longer radio wavelengths. Similar damping occurs in polar liquids, but as a consequence of the reduced intermolecular forces, the relaxation frequencies of the molecules are higher extending often within the microwave region. Water, for example, which has a high static permittivity of $\epsilon \sim 80$ and an optical one of only $\epsilon_0 = 1.8$, has a dipolar relaxation

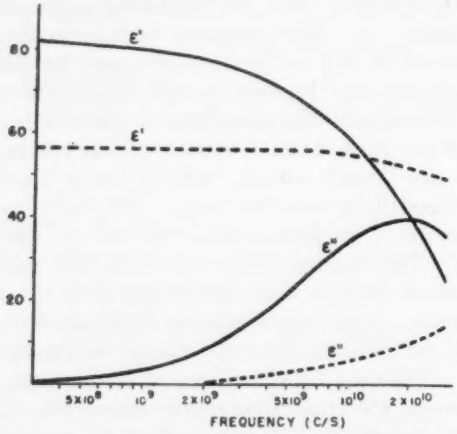



FIG. 8. Dielectric spectrum of water. Solid lines, temperature 20° C.; dotted lines, temperature 85° C.

around $\lambda = 1.6$ cm. wave length. Hence the microwave dielectric constant of water falls rapidly with increasing frequency, and its loss rises to a peak in the wave-length region between $\lambda = 1$ and 2 cm. (Fig. 8). This high dielectric loss of water molecules affects the operation of equipment and the applications of microwaves in radar and for communication purposes at wave lengths of this order.

Other liquids, whose microwave properties have been investigated, include the alcohols. These form a series—methyl ($CH_3OH$), ethyl ($CH_2O_5$), etc., whose molecules are similar to water ($H_2O$) but which are progressively larger and less polar. This causes a progressive decrease of the relaxation frequencies with increasing molecular weight, and a similar progressive decrease of the permittivity with decreasing polarity. On the other hand, symmetrical molecules, such as benzene and the paraffins, are non-polar and consequently they have only low losses at microwave frequencies.

The study of dielectric dispersion in polar liquids has so far been

used primarly for the determination of fundamental physical and chemical properties of polar molecules and polar solutions. In contrast to the microwave absorption spectra of gases, a more or less broad region of anomalous dispersion and absorption is observed instead of a discrete absorption line one. This is particularly the case in polar solutions because of the distribution of many relaxation frequencies. The microwave properties of solid dielectrics, on the other hand, are of great technical importance in connection with the design of microwave equipment, though they are only of little scientific interest.

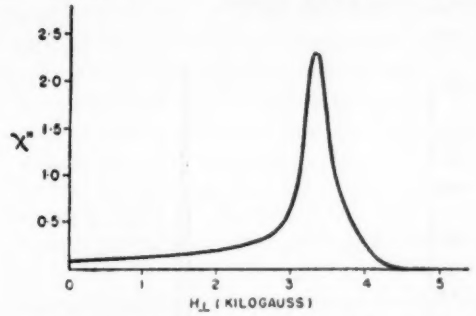



FIG. 9. Paramagnetic resonance in $MnSO_4 \cdot 4H_2O$ at $\lambda = 3.20$ cm.

**MICROWAVE RESONANCE ABSORPTION IN CRYSTALLINES**

Another branch of microwave spectroscopy is paramagnetic resonance. According to the perception of atomic theory the electrons of a paramagnetic ion possess a resultant angular momentum, and the ground state consists of a number of levels corresponding to the different orientations of the angular momentum vector in an external field. If a magnetic field of a few kilogauss is applied, the separations of these levels correspond to quanta of radiation of centimeter wave lengths. If such radiation is incident on the ion, transitions will be produced when the resonance condition $h\nu = E_2 - E_1$ is fulfilled, which can be detected by the absorption of energy from the radiation. These transitions are induced by the oscillating magnetic field of the radiation, which must normally be directed at right angles to the external magnetic field. In spectroscopic terms, we are dealing with "magnetic dipole" transitions.

Such microwave resonance absorption occurs by placing a paramagnetic salt in a cavity resonator between the poles of an electromagnet in such a way that the lines of force are everywhere perpendicular to the microwave magnetic field. The power absorbed in the cavity resonator at a particular wave length is measured as a function of the magnetic field intensity $H$, and curves similar to Fig. 9 are obtained, shown as an absorption peak, where the resonance condition holds

$$h\nu = \frac{eh}{4\pi mc} gH,$$

where $g$ is the Lande splitting factor, $h$ is Planck's constant, $e$ and $m$ are, respectively, the electronic charge and mass, $c$ is the velocity of light and $h\nu$ is the quantum of the microwave field of the frequency $\nu$.

In certain paramagnetic salts several absorption lines occur in place of a single one. These are mainly due to splitting of the magnetic energy levels, produced by the large local electric field in the crystal, caused by water molecules. This splitting of energy levels in a crystalline by internal electric fields is called the "crystalline Stark effect." Paramagnetic resonance thus provides a significant method for investigating internal crystalline fields.

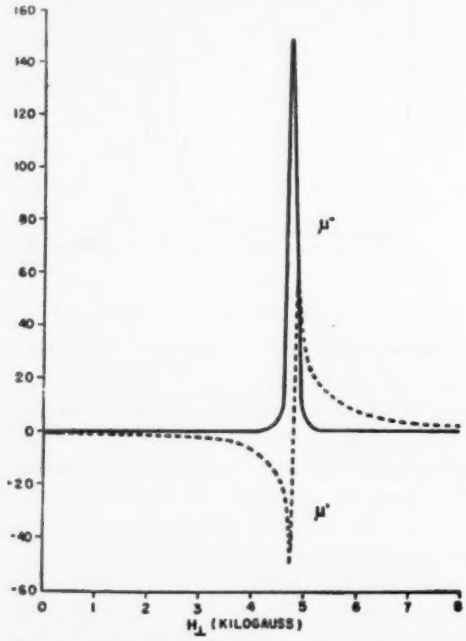



FIG. 10. Ferromagnetic resonance in Supermalloy at $\lambda = 1.25$ cm.

Similar resonance absorptions have also been observed in ferro- and antiferromagnetic metals. Ferromagnetic resonance differs from paramagnetic resonance absorption in that in ferromagnetic media the dipolar forces have a very important effect on the position of the resonance line. As a consequence of this, the simple formula above for the absorption frequency of paramagnetic salts, where the effect of the crystalline field is unimportant, is no longer valid and the formula must be replaced by a more complicated expression first derived by Kittel (24). Figure 10 shows measurements by Yager (12) of the effective permeability of Supermalloy as a function of magnetic field strength at a wave length of $\lambda = 1.25$ cm. Supermalloy is an unusually isotropic ferromagnetic material, so that the effect of anisotropy is unimportant.

Antiferromagnetic resonance absorption (13) on the other hand seems not to be investigated in sufficient detail.

### MICROWAVES IN BIOLOGICAL RESEARCH

Another application of microwaves in scientific research is associated with their biological effects. At the present time, all work on the biological effects of microwaves so far reported (14) can be explained on the basis of heat production in the extra- and intra-cellular fluids of tissues, or in the suspension media of biological-active agents when these have been irradiated *in vitro*. The heat production of microwaves in biological materials is due to loss of dielectrics which are as a rule macroscopically or microscopically heterogeneous. The most important source of dielectric loss in biological materials in the microwave spectrum is that of water molecules, due to their dipolar relaxation. Another source, which becomes more important at longer wave lengths, is ionic conductivity. In general, ionic conductivity is frequently independent up to microwaves. However, in heterogeneous materials such as animal tissues and cell suspensions, specific ionic conductivity increases with increasing frequency up to the microwave region. This effect is due to intercellular fluids being separated by thin nonconducting membranes. There is also some evidence that resonance absorption processes occur in vital molecules and in protein molecules at very short microwaves.

### ASTROPHYSICS

Since 1945 there has been much activity in the measurement of extraterrestrial radio waves coming from the sun, the stars and clouds of gas in space. To detect the very weak radio signals coming to the earth from extraterrestrial sources, a directive antenna and highly sensitive radio receiver must be used. In the microwave region a parabolic reflector is used as a directive antenna, while the receiver is frequently a radiometer using the Dicke technique (15) of measuring thermal radiation at centimeter wave lengths. Such a "radio telescope" for measuring extraterrestrial microwaves of about $\lambda = 1$ cm. wave length is shown in Fig. 11.

The radio-frequency radiation received from the sun exhibits characteristics similar to those of thermal agitation noise. In general, there are three types of solar radio-frequency radiation. First, there is a component due to thermal radiation. It remains fairly constant in intensity and is randomly polarized. The longer wave lengths of this component have their origin in the corona, which is a "black body" for meter wave lengths. At shorter wave lengths the corona becomes more and more transparent and radiation originates in deeper layers of the sun's atmosphere. At microwaves the corona is transparent and the radiation originates in the chromosphere, which is a "black body" for centimeter wave lengths. Hence measurements of solar radio-frequency radiation at different wave lengths yields information on the

electron density and the electron temperature in the sun's atmosphere. In the table below are listed the observed intensities at various wave lengths from the "quiet sun" expressed in terms of the equivalent antenna temperature. From these measurements it is seen that, over

*Courtesy of Radiophysics Division, C.S.I.R., Australia*

FIG. 11. Microwave telescope for wave lengths of about $\lambda = 1$ cm.

the wave-length range measured, the equivalent temperature of the "quiet sun" varies from 7000 degrees Kelvin at a wave length of $\lambda = 0.85$ cm. to $10^6$ degrees Kelvin at meter wave lengths (16).

| Wave Length, cm. | Effective Temperature, deg. K. |
|---|---|
| 0.85 | 6,740 |
| 1.25 | 11,000 |
| 3.2 | 16,000 |
| 10.6 | 58,000 |
| 25 | 200,000 |
| 60 | 500,000 |
| 150 | 600,000 |
| 370 | $10^6$ |

During some periods of sunspot activity there is an additional component referred to as "enhanced radiation," which is as a rule circularly polarized. This radiation has its origin in sunspots and shows a remarkable correlation with the sun-spot area, as shown in Fig. 12 for a

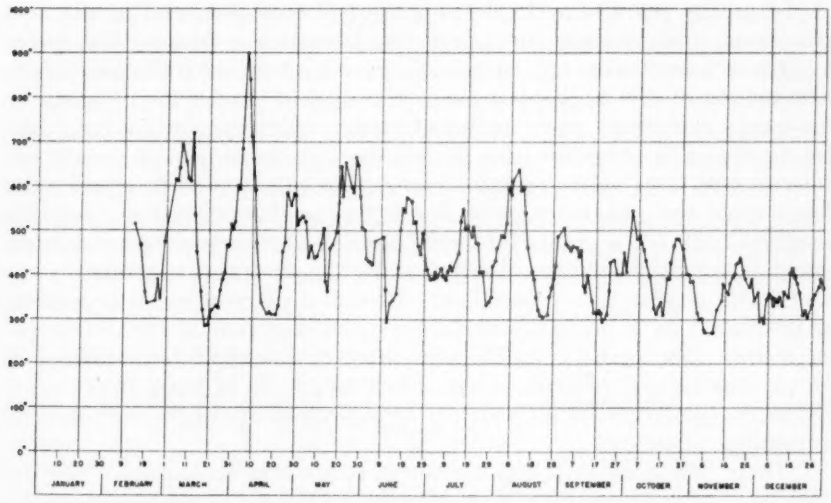



FIG. 12. The daily variation of solar intensity at 10.7 cm. wave length for the year 1947. The antenna temperature is ordinate.

wave length of $\lambda = 10.7$ cm. (17). In addition, during periods of solar activity there are sudden increases of intensity known as "bursts" and "outbursts." The intensity of these bursts decreases as the wave length of observation decreases. In the meter-wave-length region the intensity may rise to a million times that of the thermal component of the "quiet sun," but in the microwave region they are not observed at all. The occurrence of an outburst in the microwave region is as a rule coincident with the appearance of a solar flare and with a consequent sudden ionospheric disturbance known as "Mögel-Dellinger effect." It is likely that such outbursts are caused by corpuscular streams moving outward from the flares from the sun's surface. There is some evidence that the mechanism of solar outbursts is similar to the generation of microwaves by the "relativistic Doppler method" discussed earlier.

Another branch of microwave astronomy is the measurement of thermal radiation from the moon (18). In contrast to the infrared radiation, the microwave radiation from the moon originates in layers somewhat below the moon's surface. Because of this, the phase angle of the lunar microwave termperature curve lags behind the curves of surface temperatures and surface radiation. This result is consistent with the existence of a layer of dust covering the solid material of the

moon's crust. Measurements of thermal microwave radiation from the moon yield data on the structure of the moon's atmosphere and surface.

### MICROWAVES AND OPTICS

In recent years remarkable progress has been achieved in the applications of microwaves in experimental physics. Because the wave lengths of microwaves are comparable in magnitude with the geometric dimensions of the apparatus used, the optical properties of electromagnetic radiation, such as interference, reflection, refraction, diffraction and polarization can simply be demonstrated by means of microwaves (19). For example, microwaves can be used to study on a large scale the phenomenon of Roentgen-ray diffraction on crystallin gratings (20). The crystallin model studied at microwaves consists of small conducting spheres distributed in the knots of a grating geometrically similar to a crystalline. If such a macroscopical crystallin is irradiated by a beam of microwaves, the direction of the diffracted rays from this crystal coincide with those of Roentgen-rays diffracted by a microscopic crystal. Hence investigations of such microwave diffraction experiments may yield a better knowledge of the microscopic crystalline structure.

There are other relations between microwaves and physical optics which are of important scientific interest. One of the most remarkable advances made in microwave physics is the determination of the velocity of light. Such measurements had been done by Essen (21) using cavity resonators and more recently by Froome (22) using a microwave interferometer. The result obtained for the velocity of electromagnetic waves by these methods using microwaves of about $\lambda = 1.25$ cm. reduced to vacuum conditions, is $c_0 = 299792.6$ km/sec. This seems to be the most precise determination of the velocity of light ever made. Because the velocity of light in vacuum is one of the most fundamental quantities in nature, the value of $c_0$ must be known accurately for work in optics, electricity, quantum theory and nuclear physics.

### MICROWAVES IN NUCLEAR PHYSICS

One of the most significant problems in our present physical knowledge is the nature of the forces between protons and neutrons which bind nuclei and which store atomic energy. Physicists believe that these forces are associated with the creation and absorption of mesons, which particles have been observed in cosmic rays. It seems essential to produce mesons in the laboratory, so that their properties and interactions with nuclei and elementary particles can be studied under controlled conditions. This requires particle energies from a few million up to many hundred million electron volts, which can be produced by means of microwave linear accelerators.

Figure 13 shows the general arrangement of a "traveling-wave linear accelerator" (**23**). Microwaves of about $\lambda = 10$ cm. wave length, generated by means of a pulsed high-power cavity magnetron, are traveling down the axis of a tapered corrugated wave guide with a velocity a little less than the velocity of light. The waves carry a strong axial component of the electric wave field. When electrons are injected, they interact with the electric field of the traveling wave and the electrons are accelerated. By designing the corrugations properly, the phase velocity of the traveling wave is made to increase steadily, just in step with the velocity of the electrons up to velocities approaching that of light. By using wave-guide tubes about a hundred yards long it should be possible to accelerate electrons up to 1000Mev. by means of such accelerators. Microwave linear accelerators have been constructed at various laboratories and they show numerous advantages over other accelerators used in connection with research in nuclear physics.

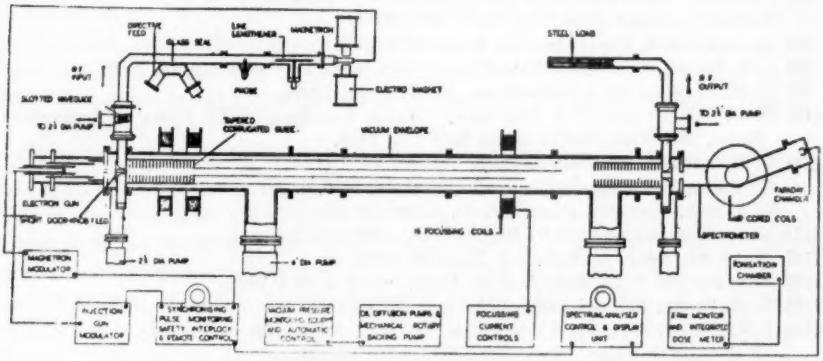



FIG. 13. Schematic diagram of a traveling wave accelerator.

### THE FUTURE OF MICROWAVE PHYSICS

There are other applications of microwaves in physics not mentioned in this paper, such as in low temperature physics and base time measurements. In the following years the region of millimetric waves will open new branches in scientific research, especially in microwave spectroscopy and biophysics. It seems to be likely that investigations in this field will lead to the use of groups of molecules to generate, detect, attenuate and filter millimetric waves. Investigations in this spectrum of the very short microwaves are of the greatest significance because quantum effects will become more and more important not only in absorption processes in molecules and atoms but also in the mechanism of interaction between electrons and electromagnetic fields as well. Hence the study of the physical properties of millimetric waves will open new aspects also to theoretical physics.

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# ON INTERACTION OF NON-LINEAR OSCILLATIONS *

BY

N. MINORSKY [1]

ABSTRACT

This paper investigates the question of interaction of non-linear oscillations. In Part I some general considerations regarding the existence, stability and certain topological rules of selection between several oscillations are outlined. In Part II these results are applied to the differential equation

$$\ddot{x} + e(x^2 - 1)\dot{x} + [1 + (a - cx^2)\cos 2t]x = 0$$

investigated by the stroboscopic method as far as the first approximation is concerned. The problem of interaction of oscillations is studied in the general case and in some special cases.

## INTRODUCTION

One of the most important properties of non-linear systems is that the principle of superposition of oscillations does not apply, so that the oscillations generally influence each other or "interact." This manifests itself in a number of experimental facts which seem to be paradoxical if considered from the point of view of the familiar linear theory of oscillations.

Thus, for example, an oscillation of a frequency $\omega$ may be either brought into play or, on the contrary, destroyed or "quenched" by setting up another oscillation of frequency $\omega'$ not standing in any rational ratio with respect to the frequency $\omega$. In some other cases oscillations of the same frequency but of different phases cannot exist together, one of them destroying the other. Very often the existing oscillation undergoes discontinuous changes in its amplitude and phase when a parameter in the differential equation passes through some special value. Likewise, the evolution of an oscillatory phenomenon considered as a function of the parameter just mentioned is different according to whether this parameter is increasing or decreasing.

In what follows we shall attempt to study the problem of the *interaction* between several non-linear oscillations, but, before undertaking this study it is useful to coordinate first the existing material in a certain logical order which will facilitate the subsequent analysis. This preliminary work of coordination of known facts is contained in Part I.

In Part II we shall investigate the problem of interaction on an example. As the procedure to be followed, we shall use again the *stroboscopic method* which has been already outlined elsewhere (1).[2]

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Other problems of a similar nature can be treated in the same manner provided certain conditions which we shall specify later are fulfilled.

Our principal interest will lie in the physical problem of oscillations and from that point of view, we shall frequently use the term "oscillator" when speaking about a certain differential equation meaning by that an oscillator represented, or governed, by the equation in question.

As is well known, the non-linear oscillatory problems are generally amenable to the differential equation of two distinct forms:

$$dx/dt = X(x, y); \qquad dy/dt = Y(x, y) \tag{1}$$

$$dx/dt = X(x, y, t); \qquad dy/dt = Y(x, y, t) \tag{2}$$

The explicit dependence on the variable $t$ (time) in (2) generally occurs under the sign of the trigonometric functions which we shall assume in what follows.

Oscillation problems of the type (1) have been much better explored than those of the type (2). It is sufficient to recall that the van der Pol equation belongs to the type (1).

In the example which we shall investigate in Part II we shall be concerned with the case when an oscillator of the type (1) is coupled to that of the type (2) through a differential equation which will form the object of our study.

In this manner it will be possible to gain an insight into the complicated nature of these oscillatory phenomena.

## PART I. ON SOME PROPERTIES OF PERIODIC SOLUTIONS ENCOUNTERED IN APPLICATIONS

### *Periodic Solutions in a Mathematical and in a Physical Sense*

One of the difficulties encountered in the modern theory of oscillations is that the definition of the term "periodic solution" always presupposes that this solution *is stable* (2).

In the linear theory, studying the oscillations near the position of equilibrium, the character of the latter, generally, determines the character of the motion occurring in its neighborhood.

In the non-linear theory, on the other hand, the motions are generally studied *in the large* to which correspond finite intervals of certain parameters and it frequently happens that the intervals of the *existence* (of amplitude and phase) and those of *stability* do not coincide. This usually results in certain complications inasmuch as the *physical existence* of an oscillation requires the simultaneous fulfillment of both conditions: (*a*) the mathematical existence and (*b*) stability.

Inasmuch as these considerations play an important role in what follows it is useful to specify them in the first place.

It is customary to study the solutions as functions of a certain parameter $\lambda$ in the differential equation. In such a case, the amplitude and the phase are certain functions of $\lambda$, say, $\rho(\lambda)$ and $\varphi(\lambda)$. The amplitude $\rho$ being an essentially positive quantity, one is interested only in

such intervals of $\rho$ in which $\rho(\lambda)$ is real and positive. As to the phase, it is sometimes expressed in the form $\sin \varphi$ (or $\cos \varphi$) $= f(\lambda)$; this, in turn, imposes a limitation $|f(\lambda)| \leqq 1$ resulting in another interval for $\lambda$. Different situations may arise according to the relative values of the four numbers specifying these intervals, say $(\lambda_1, \lambda_2)$ of the amplitude and $(\lambda_1', \lambda_2')$ of the phase. Thus, for instance, if $\lambda_1' < \lambda_1 < \lambda_2 < \lambda_2'$, clearly $(\lambda_1, \lambda_2)$ is inside $(\lambda_1', \lambda_2')$ and it is clear that both conditions of the existence are fulfilled only in the interval $(\lambda_1, \lambda_2)$ which may be called, therefore, the *interval of existence.* There may be some cases when this interval is $(0, \infty)$ in which case the amplitude and phase exist for all values of the parameter.

Similar considerations apply to the conditions of stability which generally results in the characteristic equation $S^2 + NS + Q = 0$ where $N$ and $Q$ are certain functions of the same parameter $\lambda$. The conditions of stability require the fulfillment of the two conditions $N > 0$ and $Q > 0$. This leads to the establishment of another interval, say $(\lambda_3, \lambda_4)$ to be imposed on the parameter, which may be called the interval of stability.

In this manner, the *physical existence* of an oscillation requires that there should be an overlapping interval $(\Lambda_1, \Lambda_2)$ common to $(\lambda_1, \lambda_2)$ and $(\lambda_3, \lambda_4)$. In such case the oscillation starts suddenly with a finite amplitude as soon as the threshold $\lambda = \Lambda_1$ is crossed and disappears in the same manner when the other threshold $\lambda = \Lambda_2$ is traversed. These phenomena are called occasionally "hard self-excitation."

In addition to this, some further complications appear when there exist *more than one* stable periodic solution. In this case the establishment of the oscillation according to one or the other of these two solutions is governed by a special topological rule of selection which will be outlined in the last section of Part I.

One can perceive already from these remarks that a stable stationary oscillation (or a *periodic solution in the physical sense*) is far more restricted than the periodic solution in the mathematical sense.

*Stroboscopic Method*

Since the derivation of the stroboscopic method has already been reported in earlier publications, we will not repeat it here but will limit ourselves to a few remarks.

The method is particularly useful in the case of the differential equation of the form (2) when $t$ appears explicitly under the sign of the trigonometric functions. In its present form it is applicable within the scope of validity of the method of small parameters of Poincaré and the reasons for that are exactly the same as in the latter.

Although this constitutes a limitation, the method permits extending the analysis to considerably more complicated problems than those which have been attempted so far, particularly in connection with the problems of interaction as will appear in Part II.

It is sufficient to mention briefly the results yielded by the method.

Starting with the differential equation (2) and introducing the variables $\rho$ and $\psi$ defined by the relations $\rho = r^2 = x^2 + \dot{x}^2 = x^2 + y^2$; $\psi = \arctan(y/x)$ with the corresponding relations $x = r\cos\psi$ and $y = r\sin\psi$, one obtains first these differential equations in the form

$$d\rho/dt = f(\rho, \psi, t); \quad d\psi/dt = g(\rho, \psi, t) \tag{3}$$

and, from this point, through the stroboscopic transformation of (3) one obtains ultimately the differential equation of the "stroboscopic image" of (3) in the form

$$d\rho/d\tau = F(\rho, \varphi); \quad d\varphi/d\tau = G(\rho, \varphi) \tag{4}$$

where $\varphi$ is the phase of the oscillation.

The system (4) is entirely different from (3) but there are two important connections between these two systems, *viz.*:

(*A*) The existence of a stable singular point of (4) indicates always the existence of a stable periodic solution of (3) and, hence, of (2), and

(*B*) The coordinates $\rho_0$ and $\varphi_0$ of the singular point of (4) give directly the amplitude and phase of this periodic solution.

Since the determination of singular points does not present any difficulty, a difficult problem of the determination of periodic solutions of the original equation (3) is replaced by a much simpler problem just mentioned.

The method is equally applicable to the differential equation of the form (1) as well as to the linear systems such as, for instance, the harmonic oscillator $\ddot{x} + x = 0$. In the last case the method is almost intuitive and the differential equations in terms of the variables $\rho$ and $\psi$ are

$$d\rho/dt = 0; \quad d\psi/dt = -1. \tag{5}$$

Since $\rho = x^2 + \dot{x}^2 = x^2 + y^2$, it is obvious that $\rho$ may be regarded as the total energy stored in the oscillator. In such a case the first equation (5) is nothing else but the law of conservation of energy. As to the second, it shows that the angular velocity of the representative point on the integral curve (which is a circle here) is constant and equal to *one*, the sign minus merely showing that the positive direction on this curve is in the clockwise direction. The integration of the second equation gives $\psi = \varphi_0 - t$, where $\varphi_0$ is a constant of integration, the phase. After one period $2\pi$ the angular variable $\psi$ resumes its value which shows that $\varphi_0$ remains constant.

We have found thus the well known facts which enable us to see the relation between the original $\rho, \psi$ plane and the "stroboscopic" $\rho, \varphi$ plane. In the former plane the integral curve is a circle and in the latter, it is a *fixed point* of polar coordinates $\rho_0, \varphi_0$ which corresponds to the singular point $d\rho/d\tau = 0, d\varphi/d\tau = 0$ of the stroboscopic system.

For the quasi-linear systems $\rho$ and $\varphi$ are slowly varying functions and, if they approach the limits $\rho_0$, $\varphi_0$, this shows that a stable singular point of (4) exists and this by virtue of the criterion $(A)$ shows that there exists a stable periodic solution of the original system (3) or (2). If one proceeds with the approximations of higher orders, the closed integral curves, instead of being circles as in the first approximation, exhibit the presence of harmonics but this circumstance does not destroy the periodicity of the solution.

If the system (4) has no stable singular point, the criterion $(A)$ merely asserts that there is no periodic solution of the original equation (3) but this does not rule out a possibility of some other oscillatory solution, a quasi-periodic one, for instance, (3).

*On the Multiplicity of Periodic Solutions*

It is convenient to classify the periodic solutions according to three classes of problems in which they occur: $(a)$ conservative systems, $(b)$ systems of the form (1), and $(c)$ those of type (2).

As regards $(a)$ nothing specially interesting is gained by studying the non-linear conservative systems which merely adds harmonics without changing anything in the nature of the periodic solutions. One can just as well consider the harmonic oscillator.

We have thus three cases to consider: $(a)$ harmonic oscillator, $(b)$ systems (1), and $(c)$ systems (2).

For the harmonic oscillator the solutions form a continuous family depending on two parameters $\rho$ and $\varphi$ or, which is the same, there is a double infinity of solutions. First, there exists an infinity of closed integral curves corresponding to the continuous variation of the parameter $\rho$ and for each curve of the family there is still another infinity of solutions which differ by the values of the parameter $\varphi$, the phase. This is nothing else but the well known fact that the solutions depend continuously on the two arbitrary constants of integration, the initial conditions. We have merely replaced these conditions by two other parameters $\rho$ and $\varphi$.

In the stroboscopic $\rho$, $\varphi$ plane this situation can be stated: every (fixed) point of this plane represents (or, corresponds to) a periodic solution of the harmonic oscillator.

In the case $(b)$ the solutions form a family depending on *one* parameter only. In fact, as is well known, in this case, instead of a family of closed integral curves, there exists only one closed curve—the limit cycle, at least in a finite domain, all other integral curves being non-closed (spirals) approaching this unique closed curve. However, to this closed curve corresponds still an infinity of solutions which differ only by the value of the phase. In fact, the differential equation not depending on time explicitly, it is always possible to replace $t$ by $t - t_0$, $t_0$ being an arbitrary constant, the phase, and still have a solution.

The same result can be obtained by the stroboscopic method. In fact, if one applies this method, for example, to the van der Pol equation which is of the form (1), one finds that the second equation (4) does not determine the variable $\varphi$, whereas the first equation determines $\rho$ in a single-valued manner. In the $\rho$, $\varphi$ plane, the closed integral curve is a locus of fixed points each of which represents a stable periodic solution of (1). One can also say that there is still an infinity of periodic solutions but in this case the infinity is *simple* and not double as in case (*a*).

Finally, in the case (*c*) the periodic solutions, instead of forming continuous families as in the cases (*a*) and (*b*), are isolated inasmuch as both the amplitude and the phase are now determined by the differential equation. It may happen also that there are no periodic solutions at all.

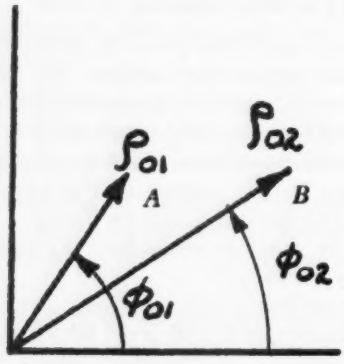


FIG. 1.

It is seen thus that, while the periodic solutions are the rule in the case (*a*) and are still numerous in case (*b*), they are rather an exception in case (*c*) and from this point of view, the stroboscopic method may be regarded as a convenient tool for detecting these exceptional periodic solutions.

*Periodic Solutions* versus *Periodic Oscillations*

We have to investigate now the case when the stroboscopic system (4) yields more than one stable singular point. The criterion (*A*) indicates that in such a case there exists more than one periodic solution and the question mentioned at the end of the first section of Part I comes to order, namely, which of these several periodic solutions will the oscillatory system "select"?

The following topological argument permits answering this question. Assume that the system (4) has two stable singular points (Fig. 1) *A* and *B* of coordinates $\rho_{01}$, $\varphi_{01}$, and $\rho_{02}$, $\varphi_{02}$. To these two points correspond two closed integral curves (circles in the first approximation)

$S_1$ and $S_2$ (Fig. 2). Since the problem is non-linear, these closed curves are necessarily isolated (of the limit cycle type), at least in a certain domain, all other integral curves being non-closed (spirals) and approaching these closed curves asymptotically.

A topological configuration of this kind is well known and is characterized by the fact that between two stable integral curves $S_1$ and $S_2$ there is always an unstable one $S'$ (4) which acts as "watershed" for integral curves if one wishes to use the hydrodynamical analogy which assimilates the integral curves to the lines of flow of the "fluid of integral curves." Moreover, the singularity at the origin of the configuration shown in Fig. 2 is considered as one such closed curve shrunk to one point. In this manner the stable and the unstable cycles always alternate.

It is obvious that a physical system represented by this configuration can be "excited" spontaneously (that is, without any artificial stimulation, by means of an impulse, for instance) only on the inner

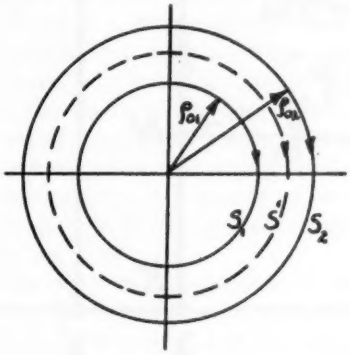


FIG. 2.

cycle $S_1$ since the "watershed" $S'$ prevents the integral curves from reaching the outer cycle $S_2$. Under this condition we can call the cycle $S_1$ as an *actual cycle* and $S_2$, as a *virtual* one. By this we mean that the physical oscillatory phenomenon will be established with the amplitude $\rho_{01}$ and not with $\rho_{02}$. This is the simplest form of "selection" between the two periodic solutions.

In reality the matter is more complicated. Very often when the differential equations contain a parameter $\lambda$ and when the latter varies, the topological configuration of solutions undergoes a corresponding variation and may, for some special or "bifurcation" values, say $\lambda = \lambda_0$, of the parameter, undergo *qualitative* changes (5). In the case with which we are concerned below such change manifests itself in the following manner. The cycles $S_1$ and $S'$ approach each other indefinitely and coalesce at the limit $\lambda = \lambda_0$ thus destroying each other. When this happens, the representative point (that is, the instantaneous state

of the physical system) which was following the cycle $S_1$, finds itself now on one of the non-closed integral curves (a spiral) of the remaining stable cycle $S_2$ establishing an oscillatory process on that cycle which thus becomes an *actual* cycle. This manifests itself by a sudden jump in the amplitude and, in some cases, in the phase of the stationary oscillation. Very often this phenomenon is called "hard self-excitation" to distinguish it from the "soft self-excitation" when no such discontinuous transitions exist.

There is still another circumstance worth mentioning. Let us represent the radii $\rho_{01}$ and $\rho_{02}$ as functions of the parameter $\lambda$ (Fig. 3) and

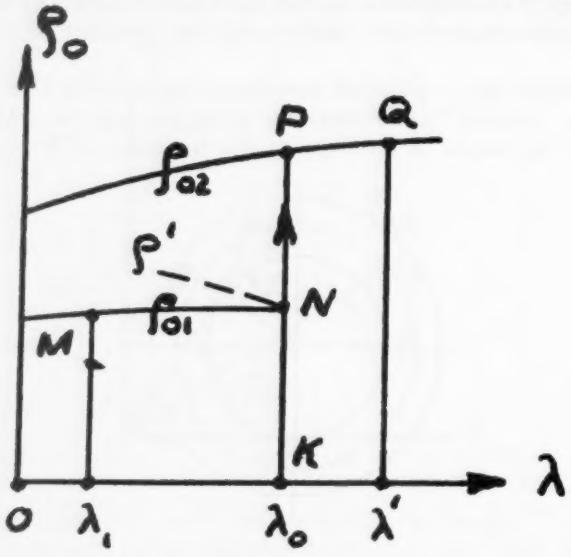


FIG. 3.

assume that the configuration of Fig. 2 exists for any value of the parameter $\lambda$, however small. If the oscillation is started when $\lambda = \lambda_1$, the cycle $\rho_{01}$ becomes the actual cycle whereas $S_2$ is only a virtual one. When the parameter increases, the arc $MN$ of the curve $\rho_{01}$ is followed. For $\lambda = \lambda_0$ the cycle $\rho_{01}$ coalesces with its unstable companion $\rho'$ and disappears. At this point there is a sudden jump in the amplitude from $N$ to $P$ and the cycle $\rho_{02}$ becomes now the actual cycle. If the parameter continues to increase the arc $PQ$ is traversed.

Suppose now that we begin to decrease the value of the parameter. At the bifurcation point $\lambda = \lambda_0$ nothing happens to the cycle $\rho_{02}$ since the oscillation had been already established on it. At this point, however, the cycle $\rho_{01}$ begins to exist, but since the motion has already been established on $\rho_{02}$, the cycle $\rho_{01}$ will be only a virtual cycle.

The essential point of this argument is in that the evolution of the

oscillatory phenomenon when the parameter is increasing is not the same as when it is decreasing.

This phenomenon is known under the name of the "oscillation hysteresis" (6) and the reason why we went into some details of this question here is because we will encounter a situation of this kind in Part II.

## PART II. INTERACTION OF NON-LINEAR OSCILLATIONS

### *Differential Equation of van der Pol-Mathieu*

As an example of the effects of interaction of oscillations, we shall investigate a differential equation of the form

$$\ddot{x} + e(x^2 - 1)\dot{x} + [1 + (a - cx^2)\cos 2t]x = 0 \qquad (6)$$

which we shall call the differential equation of van der Pol-Mathieu. For $a = c = 0$, (6) is the van der Pol equation and for $e = 0$ it becomes a non-linear Mathieu equation with a variable parametric point (7).

In order to be able to derive some conclusions, it is necessary to assume that the coefficients $a$, $c$ and $e$ are small numbers. In other words, we wish to investigate our problem when the differential equation (6) is in the neighborhood of the harmonic oscillator. In view of this it is sufficient to limit the study to the first approximation only.

These two differential equations have periodic solutions. In the first approximation the van der Pol equation has a simple periodic solution with the amplitude $x_0 = 2$ (hence, $\rho_0 = 4$) and the Mathieu equation of this form has the stationary solution $\rho_0 = a/c$, the phase being either $\varphi_0 = 0$ or $\varphi_0 = \pi$.

One can consider the differential equation (6) as representing a kind of a "coupling" between the oscillators just mentioned.

It is obvious that the only way of forming an idea as to the nature of the interaction in this case is to try to ascertain the existence of a periodic solution of (6) in the *physical sense* and, if it exists, to compare it with the known periodic solutions of the component oscillators.

It is to be noted that the term "component" here must be used with a certain degree of caution. We use this term here on a formal basis just as was mentioned in connection with cases $a = c = 0$ and $e = 0$. This term has nothing to do with the usual significance of this word used in the linear theory where it is applied to the solution of a differential equation.

### *Stroboscopic System and Its Singular Points*

We omit here the derivation of the differential system (4) corresponding to the differential equation (6) and merely indicate its form

$$d\rho/d\tau = \tfrac{1}{4}[E(4 - \rho) + (C\rho - 2A)\sin 2\varphi];$$
$$d\varphi/d\tau = \tfrac{1}{4}(C\rho - A)\cos 2\varphi \qquad (7)$$

where $A = a/\mu$; $C = c/\mu$ and $E = e/\mu$, $\mu$ being the small parameter of the two series.

$$\rho(t) = \sum_{\nu=0}^{\infty} \mu^\nu \rho_\nu(t); \quad \psi(t) = \sum_{\nu=0}^{\infty} \mu^\nu \psi_\nu(t) \tag{8}$$

which are used for the integration of (6) reduced to the form (3).

It is seen that the system (7) has two types of singular points ($\alpha$) $\cos 2\varphi_0 = 0$ (hence $\sin 2\varphi_0 = \pm 1$) and $\rho_0$ obtained by equating to zero the square bracket in the first differential equation (7) and

($\beta$) $$\rho_0 = A/C; \quad \sin 2\varphi_0 = E(4C - A)/AC.$$

The relations ($\alpha$) and ($\beta$) establish the conditions for the existence of singular points. One has to add to this also the conditions of stability which are obtained in the usual manner by forming the variational equations of the system (7). This leads ultimately to the characteristic equation $S^2 + NS + Q = 0$ where $N = (1/2)E(\rho - 2)$

$$Q = -\tfrac{1}{2}[-E(\rho_0 - 2)(C\rho_0 - A)\sin 2\varphi_0 + (C\rho_0 - A)^2 \sin^2 2\varphi_0 + \tfrac{1}{2}C\rho_0(C\rho_0 - 2A)^2 \cos^2 2\varphi_0]. \tag{9}$$

The conditions of stability, therefore, are

$$N > 0; \quad Q > 0. \tag{10}$$

The condition $Q > 0$ expresses that the singular point *is not* a saddle point and is, therefore, either a focal or a nodal point and $N > 0$ means that this singular point is stable. Conditions (10) are thus the necessary and sufficient conditions for the existence of a stable singular point of (7) and, therefore, for the existence of a stationary periodic solution of (6).

We will investigate first the conditions for the existence of a stable singular point ($\alpha$) and will postpone the investigation of singular point ($\beta$) to a later section.

We will also assume that the parameters $A$, $C$ and $E$ are positive numbers unless specified otherwise.

*Intervals of Stability; Bifurcation Values of the Parameter*

We select $E$ as the parameter of the system (7) and our problem will consist in studying the behavior of solutions of (7) when $E$ varies. As to the parameters $A$ and $C$, it will be shown that there appear certain critical thresholds according to the relative values of these parameters.

The existence of singular points ($\alpha$) corresponds to the values of $\sin 2\varphi_0 = +1$ or $\sin 2\varphi_0 = -1$ but in each of these two cases, there will appear two sub-cases according to whether $4C > A$ or $4C < A$. The case when $4C = A$ will be treated separately.

There are thus four cases to be studied: (1*a*) $\sin 2\varphi_0 = 1$; $4C > A$; (1*b*) $\sin 2\varphi_0 = 1$; $4C < A$; (2*a*) $\sin 2\varphi_0 = -1$; $4C > A$; (2*b*)

$\sin 2\varphi_0 = -1$; $4C < A$. In each case the stationary amplitude $\rho_0$ is obtained by equating to zero the quantity in the square bracket of the first equation (7).

If one attributes either negative or zero values to some of the constants $A$, $C$, and $E$ one obtains a considerable number of other cases which can be discussed in the manner similar to that outlined below. Some of these cases are mentioned later.

*Case (1a)*

In this case we have $\varphi_0 = \pi/4$ and

$$\rho_{01} = 2(2E - A)/(E - C). \tag{11}$$

The condition $N > 0$ yields here

$$E > A - C = E_1 \tag{12}$$

and $Q > 0$ reduces to the inequality

$$(C\rho_0 - A)[(C\rho_0 - A) - E(\rho_0 - 2)] < 0. \tag{13}$$

Since $E(\rho_0 - 2) > 0$, it is clear that (13) is fulfilled if $(C\rho_0 - A) > 0$ and the square bracket is negative. One verifies that the last condition is fulfilled in view of (11). As to the first, it reduces to the condition

$$E > AC/(4C - A) = E_1'. \tag{14}$$

Since in this case $4C - A > 0$, (14) yields a positive number for $E_1'$. The conditions of stability reduce thus to the inequalities (12) and (14) and, since $E_1' > E_1$, the condition (14) is a *sufficient* one for stability.

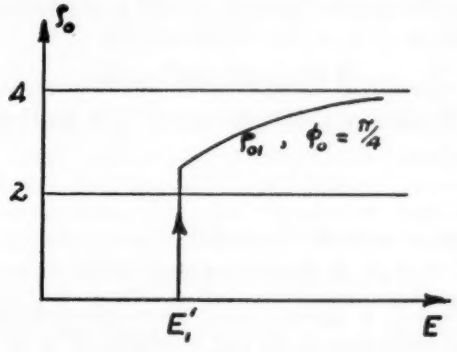


FIG. 4.

As regards the *existence* of $\rho_{01}$, we express this by requiring that $\rho_{01} > 0$. This adds two necessary conditions: (I) if $2E > A$, $E > C$; and (II) if $2E < A$, $E < C$. If one assumes, for example, the condition (I), one has to take $A < 2C$ which is stronger than $E < 4C$ which was assumed. In this case one can assert that: if $E > C$ and $A$ is in the

interval $(2C, 4C)$, the sufficient condition for the existence of a stable oscillation with amplitude (11) and the phase $\phi_0 = \pi/4$ is given by (14).

Figure 4 shows this case. If the parameter $E$ increases from a small value, there will be no oscillation until the value $E = E_1'$ is reached. At this point the oscillation $\rho_{01}$ will suddenly appear with amplitude $\rho_{01}$ given by the expression (11) in which $E = E_1'$. If $E$ continues to increase, $\rho_{01}$ approaches the value $\rho_0 = 4$ which is the stationary value of the van der Pol oscillator if it were alone (that is, $A = C = 0$). It is seen that the interaction of the two oscillators is manifested by the form of the function $\rho_{01} = f(E, A, C)$ as well as by the above specified conditions of its stability. It can be mentioned also that in this case one has what is generally called "hard self-excitation" at the point $E = E_1'$.

If one takes the case II, the arrangement of intervals is different but the discussion follows a similar argument.

We note in passing that if $(4C - A) \cong 0$, it would be impossible to fulfill (14) for any finite $E$.

*Case (2a)*

In this case $\phi_0 = 3\pi/4$; $4C > A$ and

$$\rho_{02} = 2(2E + A)/(E + C). \tag{15}$$

The condition $N > 0$ gives here

$$E > C - A \tag{16}$$

and the condition $Q > 0$ is

$$(C\rho_0 - A)[(C\rho_0 - A) + E(\rho_0 - 2)] < 0. \tag{17}$$

One verifies that the square bracket in (17) is positive in view of (15), so that the condition $(C\rho_0 - A) < 0$ results in

$$E < -AC/(4C - A).$$

This is impossible since we assume that $E > 0$, so that this case (2*a*) is ruled out.

*Case (1b)*

In this case $\phi_0 = \pi/4$; $4C < A$ and

$$\rho_{01} = 2(2E - A)/(E - C). \tag{18}$$

The condition $N > 0$ is again (12). As to the condition $Q > 0$ it is trivial since (14) reduces now to the condition $E > 0$. The condition (12) is thus a *sufficient* condition for stability. As to the existence of $\rho_0$, one ascertains that the necessary condition is $E < C$.

*Case (2b)*

In this case $\phi_0 = 3\pi/4$; $4C < A$ and

$$\rho_{02} = 2(2E - A)/(E - C). \tag{19}$$

The condition $N > 0$ yields here

$$E > C - A = E_2 \tag{20}$$

and $Q > 0$ results in the inequality

$$E > AC/(A - 4C) = E_2'. \tag{21}$$

Since $E_2' > E_2$, the inequality (21) is a sufficient condition for stability and, since there are no other conditions here, (21) is the necessary and sufficient condition for the existence of a stable stationary oscillation.

From the analysis of this section it becomes apparent that the threshold values $E_1$, $E_1'$, etc. are the *bifurcation values* of the general theory (5) inasmuch as the qualitative character of the phase portrait changes when the parameter $E$ varies as little as we please around these values.

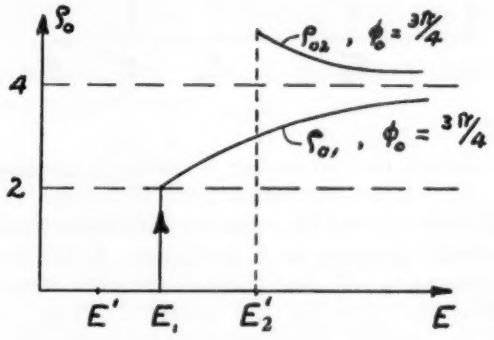


Fig. 5.

*Overlapping Intervals of Stability; Oscillation Hysteresis*

The cases (1*b*) and (2*b*) of the preceding section relate to the same condition $4C < A$ and since in both cases a stable stationary oscillation is possible, it is seen that in such "overlapping" intervals of stability the differential equations (7) yield two stable singular points. We encounter thus the situation mentioned in the last section of Part I.

In fact for $\rho_{01}$ the bifurcation value is $E = A - C = E_1$ and for $\rho_{02}$ it is $E = AC/(A - 4C) = E_2'$ and it is readily ascertained that $E_1 \gtrless E_2'$ according to the relative values of $A$ and $C$. The solution $\rho_{01}$ appears for $E = E_1$ and has the value $\rho_{01}(E_1) = 2$ at this point; since $d\rho_{01}/dE = 2(A - 2C)/(E - C)^2$, the function $\rho_{01}(E)$ increases with $E$ and approaches the value $\rho_0 = 4$ as was already mentioned. As to $\rho_{02}$, it has the value $A/C > 4$ for $E = E_2$ and decreases since $d\rho_{02}/dE = (2C - A)/(E + C)^2$.

All depends therefore on the relative values of $E_1$ and $E_2'$. This, in turn, depends on the sign of the polynomial $A^2 - 6AC + 4C^2$ which

thus establishes another limit in addition to the condition $A > 4C$ which was assumed here.

Hence, according to whether $E_1 < E_2'$ or $E_1 > E_2'$ one has two different situations depicted in Figs. 5 and 6, respectively. In fact, if

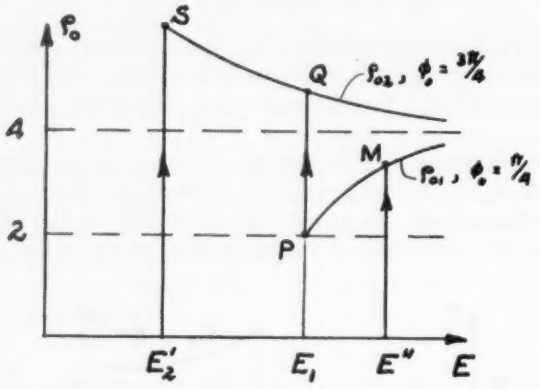


FIG. 6.

$E_1 < E_2'$ (Fig. 5) and the parameter $E$ begins to increase from a small value, say $E'$, there is no oscillation until $E$ reaches the value $E_1$ for which the oscillation $\rho_{01}$ will be suddenly established with amplitude 2 which will gradually increase as $E$ increases. If the parameter $E$ decreases the phenomenon will pass through the same stages. The essential point is that the cycle $\rho_{02}$ will remain "virtual" and cannot be converted into an "actual" cycle except artificially (for example, by a shock excitation in the region $E > E_2'$).

In the case when $E_2' < E_1$ (Fig. 6) and for the increasing parameter $E$, the cycle $\rho_{02}$ becomes "actual" cycle as soon as the value $E = E_2'$ is traversed and will remain "actual" thereafter. When $E$ reaches the value $E_1$, the cycle $\rho_{01}$ will appear, but since the oscillation has already been established on the cycle $\rho_{02}$, the cycle $\rho_{01}$ will be a virtual cycle when $E$ continues to increase. The situation is different, however, if the oscillatory system is started from some large value $E''$ of the parameter $E$. For $E = E''$ the oscillation will jump to the point $M$ of the $\rho_{01}$ curve and if $E$ decreases, the amplitude will follow the curve $MP$, the oscillation $\rho_{01}$ disappearing at the point $P$. At this point the cycle $\rho_{02}$ which was so far a "virtual" cycle will become "actual," the amplitude jumping suddenly from $P$ to $Q$. If $E$ continues to decrease the oscillation $\rho_{02}$ will follow the branch $QS$ and will disappear at the point $S$.

It is seen thus that the evolution of the oscillatory phenomenon for the increasing and for the decreasing values of the parameter $E$ is not the same and here we encounter the effect of the so called "oscillation hysteresis" (6).

*Singular Points ($\beta$); Existence of the Phase*

For this singular point $\rho_{03} = A/C$; $\sin 2\varphi_0 = E(4C - A)/AC$ as was mentioned previously. This singular point exists if $A$ and $C$ are of the same sign since $\rho_{03} > 0$. If, however, one investigates the criterion $Q > 0$ in the case when $A > 0$, $C > 0$, one finds that

$$Q = -(1/4)A^3 \cos^2 2\varphi_0 < 0$$

which means that the condition of stability is not fulfilled unless $A < 0$, in which case one must have also $C < 0$. Hence out of four possible combinations of signs of $A$ and $C$, the singular point ($\beta$) can exist only when $A < 0$ and $C < 0$. In this case the phase $\varphi_0$ for which the square bracket in the first equation (7) vanishes is given by

$$\sin 2\varphi_0 = E(A - 4C)/AC \tag{22}$$

where we write $A$ and $C$ instead of $|A|$ and $|C|$. Since $|\sin 2\phi_0| \leq 1$, the stationary phase $\varphi_0$ exists if the right hand side of (22) is less, or equal, to one.

Assume for instance that $4C < A$, then the condition for existence of the stationary state is clearly

$$E \leq AC/(A - 4C) = E_4. \tag{23}$$

The system (7) in this case is

$$d\rho/d\tau = \tfrac{1}{4}[E(4 - \rho) + (2A - C\rho)\sin 2\varphi];$$
$$d\varphi/d\tau = \tfrac{1}{4}(A - C\rho)\cos 2\varphi.$$

The condition $N > 0$ remains the same as before and the quantity $Q$ is now $Q = \tfrac{1}{4}A^3 \cos^2 2\phi_0 > 0$ which is always fulfilled. The sufficient condition for stability is thus $\rho_{03} = A/C > 2$, that is $A > 2C$ but since we have assumed $A > 4C$ the condition of stability is always fulfilled in this case.

Hence the condition (23) is the necessary and sufficient condition both for the existence of the stationary amplitude $\rho_{03} = A/C > 4$ and its stability but, since in this case $A > 4C$, the possibility of a singular point (2) is not excluded. We shall not carry out this analysis further inasmuch as it follows the same line of argument. We indicate merely the procedure. It is necessary to change the sign of $A$ and $C$ in (7) and to reproduce the calculations similar to those indicated in the two preceding sections. The only difference in this case will be in that there will be a greater complexity of the overlapping intervals of stability inasmuch as instead of two oscillations $\rho_{01}$ and $\rho_{02}$, in this case there will be also a third one $\rho_{03}$.

It is interesting to note that if $A < 4C$, $\rho_{03} < 4$ and if other conditions are fulfilled, the amplitude $\rho_{03} = A/C$ may be smaller than the other two $\rho_{01}$ and $\rho_{02}$. Under these conditions the cycle $\rho_{03}$ may be an *inner* cycle in the topological configuration of cycles and thus may

become an actual cycle at least in a certain interval. This means that in that interval the van der Pol action will be completely quenched and the Mathieu action will exist alone as if $E$ were zero in the differential equations (7).

*Critical Case; Intervals of Stability Without Interaction*

A particularly interesting case arises when $A < 0$, $C < 0$ and $|A| = 2|C|$. In such a case the singular point ($\beta$) exists with amplitude $\rho_{03} = 2$. On the other hand, since $4C - A$ is not small, the singular point ($\alpha$) also exists but in this case $\rho_{01} = \rho_{02} = 4$.

Since the singular point ($\beta$) exists only for $E < A/2 = E_3$, one has the configuration shown in Fig. 7.

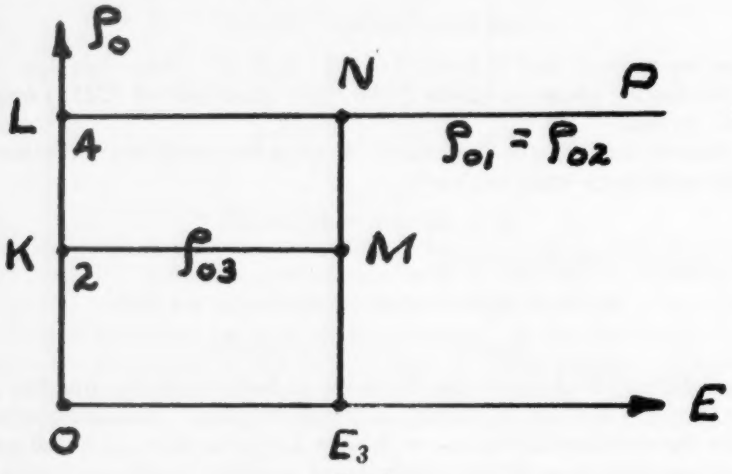


FIG. 7.

If the oscillation is started from a small value of $E$, only the amplitude $\rho_{03}$ is present since this cycle is the inner one with respect to the other two. At the bifurcation point $E = E_3$ the cycle $\rho_{03}$ disappears and the amplitude jumps from $M$ to $N$ so that the outer cycle $\rho_{01} = \rho_{02} = 4$ is now the actual cycle. The evolution of the oscillation thus follows the line $KMNP$.

If the parameter $E$ decreases, the amplitude remains constant and equal to 4 as long as the branch $PN$ is traversed. At the bifurcation point nothing happens to this amplitude inasmuch as the oscillation has been already established on this cycle but at this point there appears also the amplitude $\rho_{03}$ since this cycle is now inner in the new configuration. The phenomenon thus exhibits again the oscillation hysteresis.

It is to be noted, however, that this case, in a certain sense, is critical inasmuch as the amplitudes of the oscillation are those of the van der Pol oscillator and of the Mathieu oscillator as if they were acting alone.

Everything happens as if there were no interaction between these two oscillators.

In reality the situation is more complicated if one takes into account the phases of these oscillations. In fact, although for the singular point ($\alpha$) the amplitudes $\rho_{01}$ and $\rho_{02}$ are equal, the phases are different. In this manner there exists another bifurcation point at which the phase varies discontinuously while the amplitude does not change. This bifurcation point separates the interval of existence of $\rho_{01}$ from that of $\rho_{02}$. We shall not pursue the establishment of this bifurcation point inasmuch as this does not add anything new to what has been already said in connection with the singular point ($\alpha$). It is merely necessary to repeat the argument, replacing $A$ and $C$ by $-|A|$ and $-|C|$ and introducing an additional condition $|A| = 2|C|$.

*Special Cases*

*First Case, $C = 0$; $A \neq 0$*

In this case the differential equation (6) becomes

$$\ddot{x} + e(x^2 - 1)\dot{x} + (1 + a\cos 2t)x = 0. \tag{24}$$

This is a van der Pol equation in which the last term undergoes a parametric modulation.

The system (7) is here

$$d\rho/d\tau = (1/4)[E(4 - \rho) - 2A\sin 2\varphi];$$
$$d\varphi/d\tau = -(1/4)A\cos 2\phi. \tag{25}$$

It is seen that this system has no singular point ($\beta$). As regards the singular point ($\alpha$), we have to consider two cases according to $\sin 2\varphi_0 = 1$ and $\sin 2\varphi_0 = -1$.

For $\sin 2\varphi_0 = -1$, $\rho_0 = 2(2E - A)/E$. The condition $N > 0$ results here in $E + A > 0$ which is always fulfilled but $Q > 0$ reduces to the condition $\rho_0 < (2E - A)/E$ which rules out this case.

For $\sin 2\varphi_0 = 1$, $\rho_0 = 2(2E + A)/E$. The condition $N > 0$ is the same as before but $Q > 0$ results in $\rho_0 > (A + 2E)/E$ which is always fulfilled.

Hence (24) has a stationary oscillation with the amplitude $\rho_0 = 2(2E + A)/E$ and the phase $\varphi_0 = 3\pi/4$.

If $A$ approaches zero, the amplitude approaches the value $\rho_0 = 4$ and the phase ceases to be determined by the differential equation as it should be since (24) becomes the van der Pol equation

*Second Case, $A = 0$, $C \neq 0$*

The differential equation (6) in this case is

$$\ddot{x} + e(x^2 - 1)\dot{x} + [1 - cx^2\cos 2t]x = 0. \tag{26}$$

This is again a van der Pol equation but with a different form of modulation. The system (7) is here

$$d\rho/d\tau = (1/4)\ [E(4 - \rho) + C\rho \sin 2\varphi];$$

$$d\varphi/d\tau = (1/4)C\rho \cos 2\varphi. \tag{27}$$

Here again the singular point ($\beta$) is absent. As regards the singular point ($\alpha$), there are again two cases to be studied.

For $\sin 2\varphi_0 = 1$, $\rho_0 = 4E/(E - C)$ with the necessary condition $E > C$. The condition $N > 0$ is here $E > -C$ which is equivalent to $E > 0$ and is thus always fulfilled. As to the condition $Q > 0$, it reduces here to the inequality

$$C\rho_0[C\rho_0 - E(\rho_0 - 2)] < 0.$$

As $C\rho > 0$, the square bracket must be negative which yields $\rho_0 > 2E/(E - C)$ which is always fulfilled.

Hence, as long as $E > C$, the stationary oscillation exists with $\rho_0 = 4C/(E - C)$ and $\varphi_0 = \pi/4$. If $C \to 0$, the amplitude approaches the value 4 and the phase ceases to be determined by the differential equation since in this case (26) becomes again the van der Pol equation.

One verifies easily that the case when $\sin 2\phi_0 = -1$ does not correspond to any stable periodic solution inasmuch as the condition $Q > 0$ is not fulfilled.

There is, however, a difference between these two cases when the stationary oscillation exists, that is, between the cases

$$C = 0, \quad A \neq 0, \quad \rho = 2(2E + A)/E, \quad \phi_0 = 3\pi/4 \tag{28}$$

and

$$A = 0, \quad C \neq 0, \quad \rho_0 = 2E/(E - C), \quad \phi_0 = \pi/4. \tag{29}$$

In the case (28) for $E = A$, $\rho_0 = 6$ and in the case (29) when $E = C$, $\rho_0 = \infty$.

In other words while the differential equation

$$\ddot{x} + e(x^2 - 1)\dot{x} + (1 + e \cos 2t)x = 0$$

has a finite stationary solution, the differential equation

$$\ddot{x} + e(x^2 - 1)\dot{x} + (1 - ex^2 \cos 2t)x = 0$$

has no solution for $E = C$.

## CONCLUSIONS

The preceding analysis treats only a small part of all possible cases which arise if the parameters $A$, $C$ and $E$ are either positive, negative or zero in the various combinations.

At a first glance the problem is almost hopelessly complicated if one attempts to establish all possible cases but in reality this complication is nothing else but the result of the two additional conditions of ($a$)

stability and (*b*) topological selection. These conditions merely introduce certain restrictions on the existence of stable periodic oscillations as contrasted to that of the periodic solutions in general.

The complicated problem becomes on the contrary very simple if one wishes merely to ascertain as to what happens under a given set of the parameters $A$, $C$ and $E$, inasmuch as it reduces then to a simple consistency of two inequalities $N > 0$ and $Q > 0$.

The principal difficulty of the general problem is in that the parameter space $(A, C, E)$ does seem to yield any simple geometrical picture of the critical thresholds under the above mentioned restrictions (*a*) and (*b*).

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**"Cold" Light.**—Only one-half cent a year for electricity would be needed to light a clock face three inches in diameter with a new "cold" light adapted for this purpose by scientists at the General Electric Research Laboratory in Schenectady.

This compares with 25 cents a year required for lighting the same clock, using a one-watt incandescent lamp, the scientists pointed out. The "cold" light also is preferred because it gives off a more even illumination than do lamps. It is brighter and much easier to read than phosphorescent dials.

The new light source, known as electroluminescence, glows without producing heat, thus operating on an almost infinitesimal amount of electric power. Much of the electricity used to power other light sources goes into producing heat.

Although its glow so far has not been bright enough to illuminate a room, the new-type light is expected to be extremely useful for illuminating instrument panels for autos, boats and aircraft, as well as for lighting any size clock face, scientists said.

Ease of placing this new light exactly where you want it ranks as one of its chief advantages. To turn a sheet of conducting glass into an electroluminescent panel, a suspension of a suitable solid phosphor is sprayed on one side. Then an alternating current is passed through the phosphor coating by means of two electrodes placed on both surfaces of the phosphor.

Phosphor colors that can be produced range from orange to blue and include white.

Use of electroluminescence on clock and instrument faces is particularly desirable because a simple rheostat can be used to control the intensity of the glow. A soft, barely discernible light can be produced with the power turned low. A bright radiance is given off when the power is turned on fully.

"Cold" light was adapted for instrument and panel lighting at the G-E laboratory by Dr. Shepard Roberts, Dr. F. E. Williams, and Dr. J. S. Prener.

**Vacuum Coater.**—National Research Corporation of Cambridge, Mass., announces the Model 3111 Coater for development work and limited commercial production in vacuum metallizing. Although the vacuum evaporation of metals to provide metallic finishes on plastics, metals, fabrics, and glass has been established as a commercial process over the past ten years, process equipment has generally been limited either to laboratory apparatus or to large production units. Vacuum coating equipment has not previously been available for the transitional stage between experimental work in the laboratory and full-scale commercial production. The Model 3111 Coater bridges this gap and is designed for pilot and limited production.

The coater has a high-speed pumping system featuring the NRC Rotary Gas Ballast pump which can pump water vapor without loss of pumping speed or capacity and without contamination of oil. All necessary instrumentation is an integral part of the coater. The vertical coating chamber is 24 in. in diameter and 30 in. high. One man can easily operate the coater and the vertical arrangement of the chamber and loading tray allows short production cycles and ease of operation.

# THE CASE OF CHARGED DENSITY DISTRIBUTION *VERSUS* SEMI-PERMANENT POLARIZATION AS A BASIS FOR ELECTRET BEHAVIOR

BY

W. F. G. SWANN [1]

ABSTRACT

It is generally recognized that a distribution of volume and surface charge densities of total amount zero is the equivalent, as regards the field produced, of a suitably chosen distribution of polarization. A formal proof of this matter is given. The distribution of polarization lacks uniqueness to the extent of any contribution which, considered as a vector, is not derivable from a potential.

It is shown that, if an electret were composed of ordinary charge distributions, its effects would diminish by fifty per cent in a day or two on the basis of any reasonable assumption for the conductivity of the medium. Electret behavior must thus be based upon assumption of a semi-permanent polarization decaying in a manner independent of conductivity. The effects observed can be regarded as arising from a lag in the action of conductivity in providing for a cancellation of the effects of polarization at each instant.

The fact that the polarization, regarded as a sub-macroscopic distribution of charges, is itself not annihilated by the conductivity with a time constant appropriate thereto calls for explanation, and such is given.

Two basic hypotheses have been considered as fundamental to the operation of electrets. One assumes a polarization, semi-permanent in the sense that it varies very slowly with the time. The other assumes volume or surface charges, or both, which disappear, either by self-repulsion or by conductivity of the medium.

It is frequently recognized that, as regards external measurable effects, a distribution of charge of total amount zero is the equivalent of a distribution of polarization. The difference, however, lies in the possibilities as regards the rate of decay of the polarization or charge density. The present paper concerns itself with two matters:

A. We shall establish quite generally the fact that a distribution of charge of total amount zero is the equivalent of a suitably chosen distribution of polarization as regards the potential produced at any point. It will appear, however, that the distribution is not unique.

B. We shall show that, in general, on the assumption of any reasonable value for the electrical conductivity of a medium, electrets founded exclusively upon charge distribution as distinct from polarization would of necessity decay in their strengths far more rapidly than would correspond to the rates of decay of known electrets. The fact that a polari-

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zation may itself be considered as a system of positive and negative charges displaced in relation to each other will invite some detailed discussion as to this conclusion, but the end result will be as stated.

PROOF OF "A"

It is, of course, intuitively obvious that charge distribution of positive and negative electricity of total amount equal to zero may be regarded as the result of the separation of positive and negative from a condition in which, initially, the resulting density was everywhere zero, and in this sense, the final distribution of positive and negative electricity may be considered, at any rate roughly, as the equivalent of a distribution of polarization as regards its external effects. However, a formal establishment of the situation in more precise form is desirable.

In the case of a polarized medium, it is well known that the field, as ordinarily defined within the medium or outside of it and contributed by the polarization, may be calculated from a potential $\phi$ given in e.s.u. by

$$\phi = -\iiint \frac{\operatorname{div} P}{r}\, d\tau + \iint \frac{P_n}{r}\, dS, \tag{1}$$

where $r$ is the distance from the point at which the potential is sought to the element of volume $d\tau$ or the element of surface $dS$, as the case may be. $P$ is the polarization (moment per unit volume). The volume integral is taken throughout any region which contains the polarization, and the surface integral is taken over the surface or surfaces which bound it. $P_n$ is the component of $P$ in the direction of the outwardly drawn normal to the surface.[2]

Then, of course, if we have a distribution of real volume density $\rho$ and surface density $\sigma$, the resulting potential $V$ arising from the distributions is given by

$$V = \iiint \frac{\rho}{r}\, d\tau + \iint \frac{\sigma}{r}\, dS. \tag{2}$$

Now $\rho$ being given, let us define a vector $P_\rho$ such that

$$4\pi P_\rho = \operatorname{grad} \iiint \frac{\rho}{r}\, d\tau. \tag{3}$$

Then

$$4\pi \operatorname{div} P_\rho = \operatorname{div} \operatorname{grad} \iiint \frac{\rho}{r}\, d\tau$$

$$= \nabla^2 \iiint \frac{\rho}{r}\, d\tau = -4\pi\rho.$$

[2] The relation (1) is, of course, the origin of the well-known theorem to the effect that the polarization acts as would a fictitious distribution of volume charge density $\rho_f$ and of surface charge density $\sigma_f$ given by

$$\rho_f = -\operatorname{div} P; \quad \sigma_f = P_n.$$

Hence

$$-\operatorname{div} P_\rho = \rho. \tag{4}$$

By comparison of (1) and (2), therefore, we see that such a $P_\rho$ as is defined by (3) will give, from the volume integral in (1), alone, a contribution to $V$ equal to that which comes from the real volume charge density $\rho$. However, there remain the contributions from $\sigma$ in (2) to be considered in relation to the contributions from $P_{\rho n}$ in (1), where $P_{\rho n}$ is the normal component of $P_\rho$ at a point on the boundary. The latitude available in $P$ is through the addition to $P_\rho$ of a part $p$, satisfying

$$\operatorname{div} p = 0 \tag{5}$$

so that, subject to (5)

$$P = P_\rho + p. \tag{6}$$

We then require that

$$P_{\rho n} + p_n = \sigma. \tag{7}$$

Equation 7 would define $\mathbf{p}_n$ in terms of $\sigma$ and $P_{\rho n}$ without further comment, were it not for the fact that

$$\iint P_{\rho n} dS = -\iiint \rho d\tau; \quad \iint p_n dS = 0. \tag{8}$$

These conditions result from Green's lemma

$$\iiint \operatorname{div} \mathbf{A} d\tau = \iint \mathbf{A}_n dS$$

applicable to any vector $\mathbf{A}$. When used in connection with (4) and (5), they demand (8). Thus we must supplement (7) by the condition

$$\iint \sigma dS + \iiint \rho d\tau = 0. \tag{9}$$

This condition was contained in our original statement of the problem and the necessity of it is now evident.

If, therefore, $p_n$ is determined for each element of surface by (7), and if (9) holds, we shall have guaranteed the requirements imposed by (8).

Now, with $p_n$ determined at each point of the surface, we can determine a $p$ for each element of volume so as to satisfy (5). Indeed, we can determine $p$ in an infinite number of ways, as will appear, but any one of these ways will provide, through (6), a $P$ which will give the same potential at a point as would be given by $\rho$ and $\sigma$.

By a well known theorem of vector analysis, the most general expression for a vector $p$ is an expression of the form

$$p = \operatorname{curl} U - \operatorname{grad} \psi \tag{10}$$

where, if div $p = 0$, $\psi$ is a function satisfying

$$\nabla^2\psi = 0. \tag{11}$$

For any arbitrary $U$, the quantity $\partial\psi/\partial n$ representing the normal derivative of $\psi$ on the boundary, is given by

$$p_n - (\text{curl } U)_n = \frac{\partial\psi}{\partial n} \tag{12}$$

where the subscript $n$ denotes the normal component evaluated at a point of the boundary. Thus (12) determines $\partial\psi/\partial n$ uniquely in terms of the known $p_n$, determined from (7), and the arbitrarily assigned $U$. Again, with $\partial\psi/\partial n$ known at each point of the boundary, the solution of (11) consistent with this assignment is unique except for an irrelevant arbitrary constant which disappears in the evaluation of $p$ itself. The simplest solution of the problem is, of course, the solution in which $U = 0$ and in which $p$ is derivable from the potential $\psi$. However, since (10) yields

$$\text{curl } p = \text{curl curl } U$$

we see that with an arbitrary choice of $U$ we can make curl $p$ have any values we wish as functions of $x$, $y$, $z$, and can yet determine $\psi$ so as to make the resulting $p$, in collaboration with $P_\rho$, yield, through (6), a distribution $P$ which will simulate the effects of the distributions $\rho$ and $\sigma$.

PROOF OF "B"

Consider any point in the medium where the density is $\rho$. Let us surround the point by a small surface $S$ enclosing a volume $v$. If $E_n$ is the normal component of the resultant field at any element $dS$, the rate, $-vd\rho/dt$, at which electricity leaves the volume element is given, as far as ohmic conductivity is concerned, by

$$-\frac{vd\rho}{dt} = \lambda \iint E_n dS = \frac{\lambda}{K} \iint D_n dS,$$

where $K$ is the specific inductive capacity and $D$ is the dielectric displacement.

By Gauss' theorem, the surface integral is equal to $4\pi\rho v$. Thus,

$$-\frac{d\rho}{dt} = \frac{4\pi\lambda\rho}{K}$$

and

$$\rho = \rho_0 e^{-4\pi\lambda t/K} = \rho_0 e^{-4\pi t/kK}, \tag{13}$$

where $k$ is the specific resistance.

Taking $K = 3$, and $k = 10^{18}$ ohms/cm. $= 10^{6}$ e.s.u. approximately, as a rather large value for a wax, we have

$$\rho/\rho_0 = 1/e$$

for

$$t = \frac{kK}{4\pi} \sim 3 \times 10^{5} \text{ sec.}$$

Thus, appreciable decay occurs in three days, whereas, in the case of a good electret, a year or more can elapse before the electret loses as much as two-thirds of its strength.

It will be observed that (13) shows the decay of $\rho$ at a point to depend only upon $\rho$ itself and to be independent of the complexity of the general field in the vicinity of the point. A similar result applies to the decay of surface density, as may be proved in like fashion.

Again, according to such a picture as the above, the volume and surface densities at each point would decay according to the same law, and the strength of the electret would decay according to that law. There would be no basis for the explanation of the existence of a maximum in the potential versus time curve for the difference of potential between the surfaces of an electret such as follows so naturally[3] from the assumption of a slowly decaying "permanent" polarization operating in collaboration with an ohmic conductivity.

#### FURTHER ELABORATION OF THE FOREGOING

One might contemplate a difficulty arising from the fact that since a pure polarization is, in a sense, a distribution of charges, it might be expected to be subjected to the same law of decay through ohmic conductivity as is the case with ordinary charge density, in which case the potential difference between the surfaces of the electret—determined as it is by a combination of polarization and ordinary free charges—would decay at a rate determined by a time constant $kK/4\pi$, that is, it would decay to a fraction $1/e$ of its initial value in a time of the order of three days.

It will be recalled that the polarization theory of electrets postulates the existence of a semi-permanent (slowly decaying) polarization of very large amount which, even immediately subsequent to the formation of the electret, is almost completely annulled, as regards its power to produce a potential difference between the two surfaces, by a real charge distribution of enormous amount.[4] The effects of the total polarization, permanent and induced, are completely annulled by the charge distri-

[3] W. F. G. SWANN, JOUR. FRANKLIN INST., Vol. 250, p. 219 (1950).

[4] The mathematical details of this matter are discussed in the paper cited in footnote 3. They are also discussed, in less mathematical and more pictorial form, in a paper, "Fundamentals in the Behavior of Electrets," presented before the January 23, 1953, meeting of the A.I.E.E. in New York, and published in this JOURNAL, Vol. 255, p. 513 (1953).

butions in the above manner if initially the surfaces of the electret are at the same potential or, more exactly, if there is no resultant field in the electret. The more detailed story of this matter is as follows:

The dielectric displacement $D$, the electric field $E$, and the total polarization are related by

$$D = E + 4\pi\left[P + \frac{(K-1)}{4\pi}E\right] = KE + 4\pi P, \tag{14}$$

where $P$ is the semi-permanent polarization and $(K-1)E/4\pi$ is the induced part of the polarization.

Consider once more a small volume $v$ surrounded by a surface $S$, and let $\rho$ be the density of electricity in $v$. Then, if $E_n$ is the normal component of the field at an element $dS$ of the surface

$$-v\frac{d\rho}{dt} = \lambda \int\int E_n dS.$$

Using (14)

$$\begin{aligned} -v\frac{d\rho}{dt} &= \frac{\lambda}{K}\int\int (D_n - 4\pi P_n)dS \\ &= \frac{4\pi\lambda}{K}\left[\int\int\int \rho d\tau - \int\int\int \operatorname{div} P d\tau\right], \end{aligned} \tag{15}$$

where we have used Gauss' theorem to convert the surface integral of $D_n$ to a volume integral, and Green's lemma to convert the surface integral of $P_n$ to a volume integral. Since the volume $v$ is small, the two integrals in (15) become $v\rho$ and $v$ div $P$, respectively, so that

$$-\frac{d\rho}{dt} = \frac{4\pi\lambda}{K}(\rho - \operatorname{div} P).$$

Writing

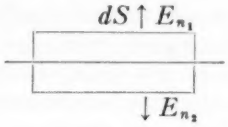
$$\rho_f \equiv -\operatorname{div} P \tag{16}$$

to call attention to the customary association of $-$div $P$ with a fictitious volume density, we have

$$-\frac{d\rho}{dt} = \frac{4\pi\lambda}{K}(\rho + \rho_f), \tag{17}$$

which tells us that if $\rho + \rho_f$ is initially positive, $\rho$ will diminish with time until ultimately $(\rho + \rho_f)$ is zero, representing a mutual cancellation of the real and fictitious volume densities. If $\rho + \rho_f$ is initially negative, a corresponding change to an ultimate value equal to zero will result.

If we draw a surface of a drum-shaped form, so as to enclose a region such as a portion of the surface of the dielectric, a region where, in general, there is a finite surface density $\sigma$, and if $dS$ is the area of the flat portion of the drum, we have




$$-\frac{d\sigma}{dt} = 4\pi\lambda(E_{n_1} + E_{n_2}),$$

where the positive directions of the vector are as in the figure.

Using (14)

$$-\frac{d\sigma}{dt} = \frac{4\pi\lambda}{K}(D_{n_1} + D_{n_2} - 4\pi P_{n_1} - 4\pi P_{n_2})$$

$$= \frac{4\pi\lambda}{K}[\sigma - (P_{n_1} + P_{n_2})]. \tag{18}$$

If the polarization is continuous across the drum-shaped area, $(P_{n_1}+P_{n_2})$ will be zero. If, however, the polarization is discontinuous—as in the case where $dS$ is on a surface which joins a polarized medium to a non-polarized medium—then $-(P_{n_1} + P_{n_2})$ represents a resultant fictitious surface density $\sigma_f$, and (18) becomes[5]

$$-\frac{d\sigma}{dt} = \frac{4\pi\lambda}{K}(\sigma + \sigma_f). \tag{19}$$

Thus, as in the case of the volume density, if $\sigma + \sigma_f$ is initially finite, $\sigma$ will change with time until ultimately $\sigma + \sigma_f$ is zero.

It will be observed that in the above, the semi-permanent polarization is not subjected to any changes on account of conductivity. Its changes are limited to its own mechanisms. In the particular case where the permanent polarization, and so $\rho_f$ and $\sigma_f$, are independent of the time, the solutions of (17) and (19) become

$$\rho + \rho_f = (\rho + \rho_f)e^{-4\pi\lambda t/K}$$
$$\sigma + \sigma_f = (\sigma + \sigma_f)e^{-4\pi\lambda t/K}$$

showing that $\rho + \rho_f$ and $\sigma + \sigma_f$ diminish exponentially with time to the value zero where $\rho$ just balances $\rho_f$ and $\sigma$ balances $\sigma_f$.

[5] In the case where one of the media—for example, the upper medium in our figure—is not polarized, $P_{n2}$ is zero and $-P_{n1}$ is the polarization in the direction of the outwardly directed normal from the surface of the medium. We observe, therefore, the customary association of such a normal component of polarization with a fictitious surface charge density.

Now what is to be said as regards the potentiality of polarization to persist in spite of the continual effort of conductivity to annul it? The efforts of conductivity in this matter are represented in the foregoing analysis by the continual tendency towards cancellation of the fictitious charges representative of the polarization by the real charges represented by $\rho$ and $\sigma$. In the case of a slowly decaying polarization, it is only because of the failure to keep "exact pace" with the decaying polarization, as a result of the conductivity not being infinite, that residual electrical effects of the electret continue to be observed as long as it has any decaying semi-permanent polarization at all.

While the last paragraph serves to illuminate the mechanism of the processes concerned, it does not answer the fundamental question which we are now raising as to why, even in the case of a polarization with no intrinsic decay, the conductivity does not annul the charges of which the polarization is composed at a rate determined by the time constant $K/4\pi\lambda$, but is content with a sort of average cancellation represented by cancellation of the fictitious charges of the polarization by real charges. In the last analysis, we are facing the problem of why the conductivity does not invade the atom itself and result in annihilation of the atomic charges. It is easy to dismiss this question as nonsensical on the basis that conductivity is a macroscopic phenomenon and is not to be thought of as having significance in the atom. However, such an evasion leaves unanswered certain questions which seem to call for an answer. It seems that the answer turns eventually upon the quantum theory itself, and raises the question of how electrical charges of opposite sign can remain apart. On the old Bohr theory, we had planetary orbits, and the holding apart of the charges was a matter easily understandable in terms of dynamical concepts. If an external charged particle sought to invade the atom with the intent of cancelling some of its charges, it was constrained, by the rules of the quantum theory of that era, to confine its activities to moving in one of the possible orbits, and a complete cancellation of any one charge by another was not included in the rules of the game. An atom with circular orbits exhibited no electrical moment as a time average. However, an elliptical orbit did provide a time average for an electrical moment, since its average distance from the nucleus is not zero. Thus, even here, we had the possibility of realizing a permanent moment, in spite of the existence of the atom in a medium which, as a whole, was conducting on account of the presence of free charged ions.

When we turn to the modern quantum theory, the story is much more satisfying. Let us first consider what we would term a real charge density, consisting of a region in which there were a number of atoms which had lost an electronic charge and were consequently in the form of positive ions. A negatively charged entity coming into the vicinity of one of these atoms would provide an electron, and this electron would

join partnership with the atom which formerly was positively charged. The resulting entity would be a neutral structure, having certainly no time average of external electrical field. In this case we have the effective annihilation of the original real charge density by the process of what we may call conductivity, as symbolized by the motion of the free negative ions.[6]

Now let us consider another possibility. It is possible for two atoms to perturb each other mutually so as to result in a structure which has a dipole electric moment. In fact, dipole moments, and indeed quadripole moments, etc., are actually calculated in the quantum theory in terms of such perturbations. In such a case, the quantum theory permits of the existence of such an unsymmetrical structure as will correspond to an entity with a moment. In a crude manner one can again picture the situation as arising by the balance of electrostatic forces by rate of change of momentum, but the picture presented by the quantum theory rests solidly upon the foundations of that theory without any such approximation. It is true that to one who views the individual charged particles as the origin of the field, the dipole moment must be considered as a time average, but even in this light it presents a satisfactory picture. Now the advent of an external charged particle is powerless to annul the moment. In general, if the atom is to be neutral as a whole, there will be no place for the invader, and it will have to wander away in search of an atom which is uncharged as a whole. Of course, our atom with its dipole moment may not succeed in retaining that moment for all time. There may be perturbations of the surrounding atoms; indeed its own internal perturbations may demand a finite lifetime for that state which is associated with the existence of a moment, and some of the structures may be continually reverting to states which have no dipole moments. In this way we can understand the gradual decay of "semi-permanent" polarization, but such decay is determined by considerations having nothing to do with the conductivity of the medium.

[6] The case where the volume density is negative can obviously be discussed in a manner which is essentially the same as the above.

**Perpetual Motion Invention.**—The hobby of Commander Ivan Monk, USN, has recently produced what has been patented as a perpetual motion machine. It is a tiny rotary-heat engine powered by atmospheric energy, and operates through small differences of temperature in its parts. In practical application, the inventor believes that it might be used to provide cheap, trouble-free power for clocks, entertainment devices, and advertising displays.

**Arctic Cable.**—All-weather, rubber-jacketed electric power cable for Arctic use has been developed by the Engineer Research and Development Laboratories, Fort Belvoir, Va.

The new military cable, which also operates successfully under tropic and temperate conditions, remains flexible at temperatures as low as −65° F. Unlike standard commercial cables which tend to act like springs when frozen, it can be uncoiled with no danger of snapping back. It does not become brittle at low temperatures nor as stiff as commercial cable.

According to Philip Mitton, Chief of the Laboratories' Rubber and Plastics Section, and Morris Smith of the Electrical Power Branch, the low temperature cable was made possible by the development of Arctic rubbers at ERDL in conjunction with the Synthetic Rubber Division of the Reconstruction Finance Corp. This development was begun after investigation of commercially available cables and rubbers disclosed that none fully met military requirements.

Experimental elastomers (the unvulcanized material with which several ingredients are compounded to produce the finished rubber) were studied to determine which offered the best possibilities. How the proportions of butadiene and styrene in the elastomer affect its cold weather capabilities and physical properties was determined. The effect of varying the temperature at which the two chemicals are combined was also studied.

As the result of extensive tests an elastomer composed of approximately 87 parts butadiene and 13 parts styrene and combined at 122 Fahrenheit was selected for cable use. It is known as GR-S 1023 and is now being used commercially to some extent where low temperature applications are involved.

Tests at the Laboratories were conducted on rubber in sheet form. After the basic elastomer had been determined, several cable manufacturers cooperated to make the compounding studies necessary to obtain rubber suitable for extruding in the forms of electric power cable.

Through government-industry cooperation large quantities of the Arctic cable are available for military application. Work on the cable is continuing to obtain improved electrical, physical, chemical and all-temperature properties and to develop improved test methods.

In the long range investigations now underway, studies are being made on the effect of long storage on low temperature characteristics; the effect of various compounding ingredients such as fillers, reinforcing pigments and plasticizers, and the effect of the state of cure. The goal is a cable which in addition to the all-weather characteristics of the new cable will give better resistance to weathering, ozone and oil.

## NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

### VIBRATION GENERATOR FOR ELECTRON TUBES

Electronic equipment is being called upon increasingly for applications where it is subjected to severe shock and vibration, and engineers have had to develop special components rugged enough to withstand such strenuous service. Electron tubes are a particular problem from the standpoint of severe vibration. In addition to being strong enough not to fail mechanically, the tube must be low in microphonics; spurious electrical signals generated by vibration of the tube must be low in relation to the desired signal that the tube is handling. In ordinary receiving tubes, microphonics may easily be a thousand or even a million times greater than the intrinsic tube noise. Tube microphonics can be a particularly difficult problem in miniaturized equipment.

An improved wide-range vibration generator is facilitating the study of low-microphonic tubes in the electron tubes laboratory of the National Bureau of Standards. Recently developed by J. D. Rosenberg, W. B. Hillstrom, and L. T. Fleming, the NBS tube vibrator produces accelerations up to 20 times that of gravity and is flat within 20 per cent over the unusually wide range of 100 to 10,000 cps. The tube under study is fitted into a hole in the vibrator's moving element, or armature. The armature is a cylindrical block of nonmagnetic material with a "voice coil"—for the audio-frequency driving voltage—at its lower end. Two flexible metal strips hold the armature centered in an electromagnetic field structure powered by 40 watts of 120-volt direct current. Although the only model of the NBS tube vibrator thus far constructed was built to accommodate subminiature-type tubes, the design could readily be modified for microphonics studies of miniature or octal tubes.

Commercially available vibration generators of the mechanically driven type have upper frequency limits of only about 300 cps.; they are primarily vibration fatigue testers, having irregular waveform and lacking the flexibility required for development work. Commercial vibrators of the electrodynamic or "loudspeaker" type are seldom useful at frequencies above 3000 cps. The moving coil in this type of vibrator is usually fastened to a drive rod that typically has objectionably sharp resonances in the neighborhood of 2000 cps.

In the NBS vibrator the upper frequency limit is raised by making the armature simple and rugged, of a material having a high ratio of rigidity to weight. Linen bakelite, which is only moderately good in

* Communicated by the Director.

this particular respect, was used for the present armature in the NBS vibrator because of its conveneience and availability. The fundamental resonance occurs in the neighborhood of 18,000 cycles; response begins to rise at about 10,000 cycles. Use of a material with a higher rigidity-to-weight ratio would substantially raise the frequency of fundamental resonance and thus the upper limit of flat response.

Development of the NBS tube vibrator was dependent upon the previous development of a small barium titanate accelerometer (NBS Model 3) capable of measuring vibration frequencies to 20 kilocycles at acceleration levels of 0.2 to 10,000 g.[1] One of these accelerometers is mounted on the armature of the vibrator to sense the vibration amplitude.

Another barium titanate accelerometer, similar in shape and weight to a T-3 subminiature tube, was developed expressly to meet needs arising in the development of the vibrator. The problem was how to design a convenient tube holder for the vibrator, sufficiently free from stray compliances and resonances to assure that the motion of the tube would conform closely to the motion of the vibrator up to at least 10 kc. The subminiature-tube-type accelerometer facilitated the design of a tube meeting this requirement.

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### FREQUENCY DEVIATION METER

To provide more reliable and sensitive receiving and recording instruments for communications signals, the National Bureau of Standards has for some time carried on a continuous program of research and development in this field. Most recent product of the program is a comparatively simple frequency deviation meter developed by Norris Hekimian of the NBS Central Radio Propagation Laboratory. The instrument indicates the deviations of a signal from a reference frequency to better than 0.5 per cent. It performs the same function as the "tuning eye" on some regular radio broadcast receivers but with sufficient precision to be used in the laboratory or as part of the production inspection procedure in a manufacturing operation.

As is true of most frequency deviation meters, the circuit of the NBS instrument consists of a limiter-discriminator arrangement and provision for driving an indicating device from the discriminator output. Normally, the arrangement includes two stages of limiting, one of discrimination, and a driving stage. However, by employing the new 6BN6 gated-beam tubes, it has been possible to reduce circuit complexity and to hold the tube complement to two in number. Advantage is taken of the input-output characteristics of the 6BN6 tube—which rises sharply to an equilibrium level, as in a step function—to obtain the second stage of limiting combined with quadrature-grid dis-

[1] "Miniature Piezoelectric Accelerometer," NBS *Tech. News Bull.*, Vol. 35, p. 141 (1951).

crimination. The indicating device—a 150-0-150 microammeter—is driven through a d-c. bridge circuit by the current resulting from an unbalanced condition in the plate circuit of the discriminator. The circuit unbalance arises when the input signal differs from the zero-set or reference signal.

To avoid plate-current cutoff by self bias, which is possible with gated-beam tubes, the NBS circuit employs 60-millihenry chokes in the signal grid returns of the 6BN6 tubes. Low-value resistors are placed in the plate leads to help dampen the plate surges and to aid in obtaining a flat limiter characteristic. A well regulated plate supply operating at 150 volts has a current drain of only 10 ma. For more stable operation, the heater supply is also regulated. Circuit components are not critical, but stable elements reduce maintenance problems. The d-c. bridge resistors in the indicator circuit are of adequate wattage and sufficiently well ventilated to avoid changes in resistance from overheating. The tank coil in the plate circuit of the first stage is set to resonate at the center or reference frequency and has a Q of about 60 when adjusted on a meter external to the circuit. The quadrature-grid tank coil also resonates near the center frequency and has a Q of 100 when measured out of the circuit. Because the quadrature tank determines the reference frequency of the deviation meter, it is enclosed to protect it from harsh handling and from being overheated by neighboring power resistors.

The NBS circuit has two inherent disadvantages: (1) Because the current flow in the grid of the first 6BN6 is limited, the input impedance of the meter varies with the level of the input signal and has a minimum value of about 10,000 ohms. This relatively large minimum is generally acceptable, but a low-output impedance driver—such as a cathode follower—may be used when necessary as a buffer between the signal source and the deviation meter. (2) Like many other discriminators, the NBS unit shows considerable interaction between discriminator adjustments. This often requires a tedious and prolonged initial aligning procedure; however, the operation need be performed but once when the meter is to be used at one fixed frequency.

Briefly, the alignment procedure is as follows: (1) The indicating meter is replaced by a high-impedance voltmeter, and the direction of the needle swing is noted for increasing input signal from 0 to 1 volt. The plate coil of the first stage is then tuned for maximum voltmeter reading in the noted direction. During this period, the input potential is maintained at the lowest value that will allow readable output changes. (2) The zero control of the d-c. bridge and the tank capacitor in the quadrature grid are adjusted simultaneously for approximately a normal discriminator curve as the signal generator frequency is slowly varied about 10 kc on either side of the desired mean frequency. (3) With the input signal set at the desired mean frequency, the input

level is slowly varied from about 0.2 to 2.0 volts, and the cathode resistor in the output stage is adjusted for the flattest limiter characteristic obtainable. The signal generator frequency is monitored continuously with a stable receiver and beat oscillator to insure that the frequency does not vary when the output is changed. (4) The high-impedance voltmeter is replaced by the indicating meter, and steps (2) and (3) are repeated to obtain the desired symmetrical discriminator characteristic and good limiting.

Calibrations may be performed in several ways depending on the equipment that is available. When a well calibrated signal generator may be used, it is necessary to calibrate only the output indicating meter directly against the input frequency of the signal generator. The input to the deviation meter should be set to at least 2.0 volts during the calibration.

One of the NBS frequency deviation meters was used during a 1600-hour continuous life test to determine the difference between the local and signal oscillators in two 30-Mc receivers. At the conclusion of the examination the deviation meter showed a change of less than 200 cycles, which was probably due to changes in the crystal oscillators of the receivers. No adjustment to the deviation meter was necessary after the initial calibration.

NOTE: For additional information, see "Frequency-Deviation Meter Plots Drift," by N. C. Hekimian, *Electronics*, Vol. 25, pp. 6, 134 (1952).

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The following papers will appear in this JOURNAL within the next few months:

COULSON, THOMAS: The Story of Domestic Lighting.
LANGHAAR, HENRY L.: The Principle of Complementary Energy in Nonlinear Elasticity Theory.
BERGER, W. J. AND EDWARD SAIBEL: On the Inversion of Continuant Matrices.
LEE, J. F.: The Thermodynamics of Critical Phenomena in Gases.
KU, Y. H.: A Method for Solving Third and Higher Order Nonlinear Differential Equations.
LIVERSIDGE, HORACE P.: Industry's Responsibility in the Shortage of Engineers.
KLINGER, H. H.: Radio Astronomy.
MOON, PARRY AND DOMINA EBERLE SPENCER: TEM Waves in Cylindrical Systems.
PAI, S. I.: On Turbulent Flow in Circular Pipe.
CUNNINGHAM, W. J.: Amplitude Variations in a Frequency-Modulated Oscillator.

## NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

***Abstract of* Effects of Sonic Vibration on the Proteolytic Activity of Pepsin.**—GEORGE M. NAIMARK AND WILLIAM A. MOSHER (*J. Acoust. Soc. Am.*, **25**, 289 (1953)). Pepsin preparations were sonically treated for various periods of time in a 9-kc magneto-striction oscillator with the temperature maintained at 13° C.–16° C. during treatment. After sonic treatment the residual proteolytic activity of the enzyme was determined by measuring the turbidity decrease during the digestion of an albumin substrate. It was found that dilute solutions (0.2 per cent) of Merck U.S.P. pepsin were rapidly inactivated by sonic treatment whereas highly concentrated solutions (10 per cent) were refractory to ultrasonic destruction. Sonic irradiation of a pure Armour crystalline pepsin solution (0.013 per cent) yielded slight enzyme inactivation only. In no instance was enzyme activation observed in this study.

## NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, EXECUTIVE DIRECTOR

### NAVAL AVIATION EXHIBIT

The Aviation Supply Office is among the most recent sponsors of a Museum exhibit. Constructed at little cost by using display items on hand or in stock, the exhibit contrives to present in a most attractive manner the scope and importance of one of the Navy's shore installations.

The center background of the exhibit demonstrates the part played by the Aviation Supply Office in the Navy's supply system. It shows graphically that this department is the nerve center of the procurement and distribution of aviation spare parts and supplies for the Navy. Under the command of Rear Admiral Ralph J. Arnold, Supply Corps, U.S.N., the office deals with a bewildering array of over 400,000 aviation items. Distribution of these items is controlled worldwide by the Aviation Supply Office from their depot in the northeastern section of Philadelphia, for the purpose of meeting all requirements while maintaining the lowest adequate stock consistent with aircraft operating needs. This involves the consideration of manifold factors, such as usage, life of aircraft, interchangeability of parts, and a multitude of other factors.

The great bulk of the work performed by the Supply Office would scarcely be of interest to the layman, but a judicious selection of those items which appeal to the imagination of anyone interested in the well-being, safety, and feats of our airmen has permitted the organization of this attractive exhibit.

The exhibit features a Pratt and Whitney R2800 reciprocating engine used in aircraft. To make the operation of this intricate piece of mechanism more comprehensible to the visitor, the engine has been cut away and is push button operated.

In keeping with the aviation theme large photo murals have been used to provide a background. These striking pictures show planes preparing for the take-off, pilots approaching their planes aboard an aircraft carrier, the interior of a plane in flight, and mechanics repairing engines at an airstrip. These large action pictures form an appropriate background for the display of Navy survival equipment. Included in this is a seven man life raft and all the equipment necessary to keep the crew alive until rescued. From the material shown the visitor must be impressed with the care and ingenuity devoted to the development of those devices which aid in promoting the survival of aircraft crews.

Also on display are two mannequins dressed in the latest flight clothing and assorted cut away spare parts which are but a small selection from the many thousands of items stocked by the Naval Aviation Supply Depot.

Rear Admiral Ralph J. Arnold, USN, Commander Aviation Supply Office, presses the button starting the cut-away model of the aircraft engine in the Aviation Section.

(Left to right: S. Wyman Rolfe, President of The Franklin Institute, Rear Admiral Arnold, Dr. Henry B. Allen, Executive Vice-President of The Franklin Institute, and Rear Admiral John H. Brown, USN, Commandant, Fourth Naval District.)

One of the most interesting features of the display was, unfortunately, only on temporary display. This was the "Gorgon" guided missile, a ram jet powered pilotless aircraft which travels at over 650 miles an hour as high as 7 miles in the air. Launched high above the ground from a "mother" plane, the missile is radio-controlled by either the "mother" plane, from the ground, or by a ship.

This exhibit is the forerunner of a series of exhibits which is being planned for later display by the Naval Aviation Supply Office.

## NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

### MEASURING FRICTION AT HIGH SPEEDS

BY

WILLIAM W. SHUGARTS, JR.

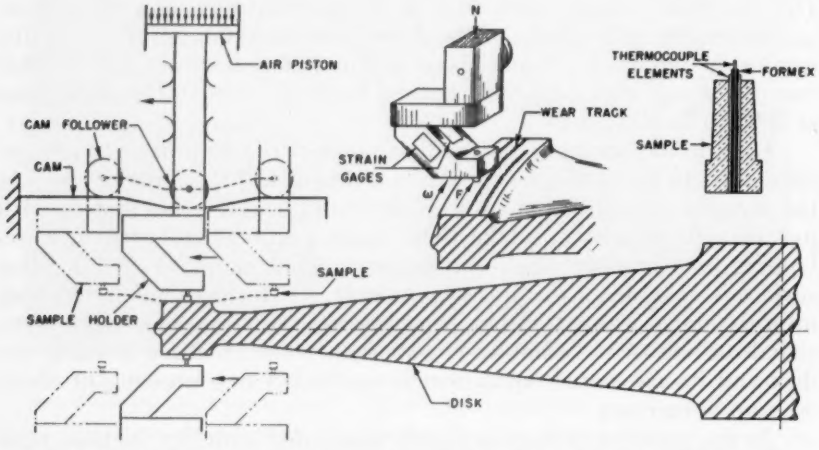
Recent changes in the instrumentation associated with the High-Speed Friction Testing Machine show promise of revealing rapidly fluctuating friction phenomena with a resolution heretofore impossible. This machine, designed and built by the Laboratories staff some years ago under the sponsorship of the Army Ordnance Corps, produces dry friction between solid materials at sliding velocities from 150 to 2000 fps. It is capable of applying normal loads of 10 to 100 lb. (pressures of 2000 to 30,000 psi.).

The friction occurs when small samples of the test material are applied to both faces of the rim of a rotating disk 2 ft. in diameter. As the samples are applied at the rim they are given a radial motion with just enough velocity to make them trace a spiral path without overlapping on successive disk revolutions. Thus a non-retracking test surface is possible for distances up to 40 ft. For tests where retracking information is wanted, either from one test or from successive tests, radial motion of the samples is not used. The chamber housing the disk and its associated equipment is evacuated to a pressure of about 30 mm. of mercury.

As the problem of wear is closely associated with dry friction, wear measurements are made of the change in linear dimension of the sample as well as its change in weight. Also intimately connected with both friction and wear is temperature, particularly the instantaneous temperature at the mating surfaces. When a thermocouple is used to measure the rapidly fluctuating temperature at the friction surface of a sample whose diameter is approximately 0.100 in., care must be taken if the temperature readings are to be reliable. Faithful response to rapid changes in temperature can be obtained only with a couple whose size is a minimum. The small size is also necessary to minimize the contamination of the sample surface by thermocouple materials. If possible, the choice of these materials should include at least one that is thermoelectrically similar to one of the two friction materials. In addition, from the work of others, it appears that a temperature gradient may exist across the surface of the sample, hence the smallest thermocouple possible must be placed at the proper location, if peak tempera-

tures are to be recorded. For these reasons a coaxial thermocouple, with an outside diameter of 0.022 in., is centered in the sample. Whenever possible one of the thermocouple materials is chosen to approximate the material under test. Moreover, the junction of the couple is made by contact with the friction disk instead of by the usual weld.

Both frictional and normal forces are measured by electrical resistance-type strain gages bonded to the sample holder. These gages provide signals proportional to the respective forces with a frequency response above 50 Kc. They are calibrated statically by the use of weights. The load during test is applied to the sample holders by a combination of air pistons and cams. Air pressure on the pistons keeps the sample holders against the cams until, at the time of traversing the rim of the disk, the sample holders are let down at one edge of the rim




Details of the load-application system, force measuring gages and coaxial thermocouple.

surface and are picked up as they reach the other edge. During the test the sample holders ride free and assume the full load of the air pistons.

Formerly, instrumentation had been used to amplify and record the three chief variables—frictional force, normal load, temperature—and a 1-Kc. time signal, which were photographed from the face of a cathode ray oscilloscope. The simultaneous presentation of the four signals on a single oscilloscope was accomplished by means of an electronic switch whose switching rate was 100 Kc.

Two difficulties became apparent as more knowledge of high speed friction was acquired. The frequency of fluctuation under some conditions of load and speed exceeded the capabilities of the instrumentation. The over-all frequency response was approximately 3 Kc. at best and

the single frame photography limited the resolution of even these highly attenuated signals.

The present improved instrumentation incorporates amplifiers, a four-channel oscilloscope, and a drum camera. The amplifiers respond to anything from direct current to 1 megacycle with a maximum gain of 5000. The four-channel oscilloscope has a similar response up to 100 Kc. on each channel with a maximum gain of 120. The drum camera is able to resolve frequencies above 50 Kc. with a maximum speed of 125 fps. on a 47-in. length of film. The over-all response of 50 Kc. is compatible with the response claimed for the strain gages by the manufacturer. In addition to the three signal channels—frictional force, normal force and temperature—there is a time channel which can present sharp pulses at tuning-fork controlled frequencies of 1, 3, 5 or 9 Kc. The choice of frequencies is dependent only upon film speed so as to provide for convenient reduction of data from the film.

In a study of friction and wear it is impossible to over-emphasize the importance of surface cleanliness and, to some extent, surface finish. Past practice on this test machine included the installation of a special surface grinder that allowed frequent finishing of the disk rim surfaces. These have been reproduced with a roughness that lies between 2 and 4 microinches (rms.). In addition to grinding the surfaces, other finishing techniques include rubbing with 120- and 240-grit emery cloth. The cleaning techniques include swabbing with cheese cloth soaked in alcohol, as well as wiping with dry cheesecloth. Most of these past practices will be used for the first portion of the test program with changes made where dictated by experience or by additional knowledge from other sources.

Present plans call for not only the accumulation of data on friction and wear at the speeds and loads indicated, but also such intriguing problems as a study of the stick-slip phenomenon and its relation to the natural frequency of the sample holder and to the surface finish of the disk. The work of others has been chiefly at speeds less than 150 fps., hence, the extension of the knowledge of stick-slip into a higher speed range should prove rewarding.

## BOOK REVIEWS

STOCHASTIC PROCESSES, by J. L. Doob. 654 pages, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $10.00.

The transition from the classical interpretation of physical laws to modern statistical concepts found its expression early in quantum mechanics and the uncertainty principle. Proponents of the new thesis struggled at once with the restatement of laws and the requisite mathematical statistics. When the application of probability theory to time dependent processes came into its own, the term "stochastic process" was used to describe this class of relationships. It has since been found to apply to almost all branches of science.

An increasing number of authors quickly discovered that mathematical measure theory gave the precise setting for the needed probability statements. Judging by the large number of papers published each year it was evident that a formal exposition of the general theory of stochastic processes would have to follow sooner or later. Niels Arley published a specialized study on stochastic processes applied to cosmic radiation in 1943 and the need for a text on a high mathematical plane became even more apparent. The second Berkeley Symposium in 1950 included a paper by J. L. Doob announcing that his long-awaited volume was finally in preparation. Thanks to the commendable support of the Office of Naval Research, "the infinite time required to write the book" has been reduced to two years. The fruits of Professor Doob's labors are at last in the hands of the public.

The author has stressed "those parts of the subject which appeal most" to him. In view of the overwhelming scope of the volume, we can but admire the keen insight which the author has, both into theory and applied fields. Needless to say, he has illuminated this extremely complex subject with unusual intensity. The first draft of the manuscript was written for those who had mastered the results of the mathematical theory of measure. But editorial reviewers persuaded the author to include important theorems from that mathematical discipline and the resultant supplement is a gem. Without the burden of proof, it develops the concepts in a form palatable to even the most sophisticated reader. The decision to include this supplement is laudable. The results should allay the author's fears that this constitutes an "inconsistent compromise."

The body of this text comprises twelve equally strong chapters, devoted to a variety of stochastic processes. The important measure theoretical aspects of probability theory are developed in the first chapter, the nature of stochastic processes in the second. The author then goes on to discuss the aspects of processes with discrete and continuous parameters. In addition to an elaborate treatment of Markov processes we find here a complete exposition of the theory of martingales. The English speaking world will be in debt to the author for this chapter. He was the first to draw attention to this concept through his Symposium paper. Its importance in the development of relevant theorems is reflected by the elegance and dispatch with which fundamental theorems for many processes are derived from martingale theory.

It is virtually impossible to give here a complete account of this superb volume. Touching upon some of the highlights only, we find in Chapter VIII on Processes with Independent Increments restatements of Brownian Motion and Poisson Processes. Chapter IX elaborates on processes with orthogonal increments. The following two chapters treat stationary processes, again separating discrete and continuous parameters. The final chapter outlines the specialized problem of prediction theory and gives results for regular and deterministic processes in terms of measure theoretical language.

Each chapter of this important addition to the Wiley Publications in Statistics supplies a well-balanced diet of theorems, proofs, and suggested fields of applications. The sophisticated reader will be delighted to find compact and precise restatements of familiar

theorems in the language of measure theory. The scope of the volume excludes nonstationary processes and it is hoped that the author will find time to treat them with equal elegance. Although no "formal" problems are included, the reader finds plenty of "elementary" work left. The publishers, too, have put every effort into making the highly mathematized pages as attractive as possible: the formulas are easy on the eye and are well spaced on high quality paper.

Because his subject is of utmost importance in all sciences which permit of probabilistic interpretation, Professor Doob's classic is a "must" in the hands of mathematical and applied statisticians. It is as important a contribution on the most advanced level as any serious researcher could wish to possess.

CARL HAMMER

THE SCIENTIFIC ADVENTURE. ESSAYS IN THE HISTORY AND PHILOSOPHY OF SCIENCE, by Herbert Dingle. 372 pages, 14×22 cm. New York, Philosophical Library, 1953. Price, $6.00.

For some time there has been a noticeable coolness between the scientists and the philosophers and a tendency among the former (especially those who value the things that catch the eye and have a price tag) to deplore that, as their colleagues grow older, they have a tendency to depart from true science to embrace mere philosophy. Surely the time has spased when both scientists and philosophers should have realized that no distinction exists between science and philosophy. Science, as a separate self-contained study, dates from the seventeenth century. Before that time, such consideration as was given to the subject matter of present-day science was given it by philosophers, and regarded as a part of their philosophizing. When, in the seventeenth century a new kind of procedure was introduced, it was looked upon by its pioneers not as an attack upon a new problem, but as a new attack upon an old problem. These pioneers were not called scientists but natural philosophers, which is the more correct term.

The attitude of those who persist in raising a distinction between science and philosophy fosters the belief that science, by its nature, is limited to those problems in which it has been successful. Thus there has grown a conception of science which appears to originate more in the accidents than in the substance of its being. Such definitions of science as that uninspired one by Eddington that the cleavage is not "between the concrete and the transcendental but between the metrical and the non-metrical" can be mischievous, because they are not based upon the fundamental consideration of the possibilities inherent in the "scientific method," but rather upon its actual achievements. According to such a doctrine those ancients would have been correct who asserted that men would never be able to fly because the principles of navigation were only applicable to sea and land travel.

Science, although a genuine philosophy and not an upstart activity, cannot prove to be a universally satisfying philosophy. For all its impregnable basis and the stability of its superstructure, it is always essentially unfinished.

A new approach to a broader outline is to regard science as a part of philosophy—one aspect of the desire to make sense of our experience, to see it as an ordered rational system instead of a succession of fortuitous happenings. This new approach is being pursued more diligently in Britain than in this country. Herbert Dingle, professor of History and Philosophy of Science, at University College, London, and who is author of an earlier book *Through Science to Philosophy*, presents in his latest volume nineteen essays designed to demonstrate the results of applying the doctrine of the new approach. Disclaiming the right to be called a historian, he applies his viewpoint to the past; to the work of men like Copernicus, Galileo, and Thomas Wright; and to past and present theories on the universe. The second half of the volume is devoted to essays on the philosophical viewpoint of the scientist, on the laws of nature, on the theory of measurement, and on science and ethics.

The book will be welcomed by every scientist who is willing to look up from his work and to notice where he is going. There are few people as well qualified as the author to get beneath the surface of modern science and to form a trustworthy idea of its essential nature and possibilities. Professor Dingle has produced a volume of absorbing interest and lasting value. The book is worthy of the highest recommendation. T. C.

THE THEORY OF HOMOGENEOUS TURBULENCE, by G. E. Batchelor. 197 pages, diagrams, 15×22 cm. New York, Cambridge University Press, 1953. Price, $5.00.

The title of the book is highly misleading since only the turbulence in an incompressible fluid is treated. As is known, some amount of theoretical and very little experimental investigation in the turbulence of a compressible fluid has been done. There is nothing said about that. Some parts of the book make a little odd impression. In the introduction the author says that it is his belief that applied mathematics is a science in its own right, and is neither a watered-down version of pure mathematics nor a prime form of physics. This is obviously not true since applied mathematics is a branch of mathematics and certainly this book does not belong to this field of science. Perhaps the author had in mind "theoretical and applied mechanics," where certainly his book may be properly located. Using the author's words, "the manner of presentation of the material in this book has been chosen, not with an eye to the needs of mathematicians or physicists or any other class of people, but according to what is best suited, in my opinion, to the task of understanding the phenomenon." And below, he states that such an approach "is consistent with my view of the nature of the subject-matter." Briefly, the readers are for the author and not the author for the readers. Certainly, the reader would welcome a little more consideration from the author to facilitate for him the entrance into the intriguing but complicated domain of turbulence. Being consistent, in a few parts of the book the author passes with great speed, sometimes in the span of a few pages, from the domain of mathematics into that of physics, next into that of engineering and back again to mathematics in the same order, or in a reverse one. The reader feels like those "big whorls having little whorls, which feed on their velocity," etc. in Richardson's rhyme on Kolmogoroff's hypothesis. Turbulence is the domain which requires a collaboration of more than one branch of the applied sciences. Consequently in a methodical and systematic approach to the problem of explaining turbulence, a book of this kind should be divided into a few distinct parts: mathematical theory, physical part, engineering tests, etc. In each subsequent part continuous references should be made to the previous ones. Although the volume of the book would increase, a clear, presentation of the matter would result. From this standpoint a few parts of the book give the impression that somebody was in a big hurry and put together many pieces of mathematics, mixed with physics and engineering. With a little more effort the author could present an excellent book.

Much of the research on the subject has been done at Cambridge by the author and his colleagues. Very little attention is paid to the work done in prominent research laboratories in other countries, as for instance, in a few centers in the U. S. A. This is another odd feature of the book: an interest in "domestic" turbulence and almost complete disregard for the "international" status of this phenomenon. The reader certainly would like to know more about the "world situation" on that subject. After these preliminary remarks, let us discuss individual sections.

Chapter I, Introduction, contains a description of turbulence, its various kinds, mathematical formulation of the problem, and a brief history of the subject. In this section the author argues that it is justified to ignore the molecular structure of the medium and regard it as a continuous fluid. Briefly, he restricts himself to dense gases only, since definitely Navier-Stokes equations are not adequate enough to describe the status of turbulence in rarefied gases.

Chapter II describes the method of taking averages, the complete statistical specification of the field of turbulence, mean values of velocity products, general properties of the velocity correlation and spectrum tensors and Fourier analysis of the velocity field. The applicability of ergodic theory is assumed; thus a time average is identical to a probability average for the experimental field.

Chapter III deals with the kinematics of homogeneous turbulence: the velocity correlation and spectrum tensors, the vorticity correlation and spectrum tensors, symmetry conditions and isotropic turbulence.

Chapter IV contains discussion of some linear problems: a simple harmonic oscillator subject to a random force, passage of a turbu-

lent stream through wire gauze, and the effect of sudden distortion of a turbulent stream. The examples are of interest for their own sake "as a demonstration of how completely—and easily—a problem of turbulent motion may be solved when the relevant equations are linear." Certainly, there is nothing new in this statement.

Chapter V deals with the general dynamics of decay. It is in some parts of this chapter and of some subsequent ones, where the reader must stretch his mind from mathematics through physics up to engineering and to stay in this status for the time of reading. The author describes methods of using the Navier-Stokes equation, the flow of energy, the permanence of big eddies, the final period of decay, and the dynamic equations for isotropic turbulence.

Chapter VI contains the universal equilibrium theory: the hypothesis of statistical equilibrium, turbulent motion at a large Reynolds number, the hypothesis of independence of Fourier components for distant wave-numbers, the universal equilibrium, the inertial subrange and the energy spectrum in the equilibrium range. The book is full of statements which seem to be challengeable, controversial, and certainly open to endless discussions. In this chapter, the author admits that "it became necessary to supplement the deductions from the basic equations with inferences from measurements; by arguing both forwards and backwards it was possible to obtain an understanding of one aspect of the turbulence which is unlikely to be wrong in general principle."

Chapter VII deals with the decay of the energy-containing eddies: the decay of total energy, evidence for the existence of a unique statistical state of the energy-containing eddies, the quasi-equilibrium hypothesis, the equilibrium at large wave numbers for moderate Reynolds numbers and Heisenberg's form of the energy spectrum in the quasi-equilibrium range. In this chapter the process of the order of reasoning, applied in the previous chapter, is carried still further and the author is obliged to rely to a considerable extent on the empirical evidence.

The last chapter (VIII) deals with the probability distribution of the velocity or its derivatives at one point, and with the joint-probability distribution of the velocity at two points: the experimental evidence, the hypothesis of a normal distribution of the velocity field associated with the energy-containing eddies, determination of the pressure covariance and the small-scale properties of the motion.

The bibliography contains an extensive list of references on the turbulence in incompressible media only.

A few imperfections, mentioned above, seem to be justified by the fact that the book is the first of its kind to be published. Certainly, it is a valuable addition to libraries of research laboratories on turbulence in particular and on fluid dynamics in general. It is written in one of the oldest centers of turbulence investigation and clearly reflects the opinions of the scientists at Cambridge on the problem of turbulence. The great number of problems discussed, touched and sometimes only scratched on the surface, adds to the great interest which the reader finds in this branch of fluid dynamics. The great number of controversial problems of a fundamental nature, intriguing by their attraction to certainly not brief discussions, adds to the promise of many investigations in the future. The restricted volume of this review does not allow the reviewer to present even a small quantity of these interesting problems. The book represents the first volume of the Cambridge monographs on mechanics and applied mathematics. It is nicely printed but it is an open question whether in its present form of arguing both forwards and backwards and of the lack in some important parts of it of a methodical systematic approach to the solution of problems in question it should be recommended unconditionally as a textbook for graduate courses. But certainly, with the proper interpretation and perhaps some adjustments here and there, and with proper formulation of the controversial parts of the subject it may serve as a guide and render valuable services.

M. Z. v. KRZYWOBLOCKI

GLYCEROL, edited by Carl S. Miner and N. N. Dalton. ACS Monograph 117. 460 pages, diagrams, 16×24 cm. New York, Reinhold Publishing Corp., 1953. Price, $12.00.

In the past twenty years, the technology in many fields has advanced greatly due to the ever increasing demands of production.

Glycerol, its production, refining, and use most definitely fit into this category. There have been great improvements in the syntheses, production techniques, and refining of glycerol since about 1930, and World War II gave added impetus to the growth of all of these phases. These new achievements are either hidden in industrial practice or in scattered publications. The editors have used this book to compile a modern glycerol technology. The book contains all of the old established or so called classical work as well as the most recent advances in the field presented by specialists from the particular branches. The presentation of the recent (within ten years) high yield syntheses of glycerol from propylene and by interesterification of low grade fats will be of special interest to the glycerol producers. The use of ion-exchange resins in refining glycerine is also discussed including the most recent technique of mixed bed deionization for purifying the product.

The book is divided into eleven chapters covering the following subjects: history and economics of glycerol; its natural source and occurrence; methods of production, recovery, and refining; *Pharmacopeia* standards and government specifications of the United States and Great Britain; analytical methods; physical properties of glycerol and its solutions; chemical properties and derivatives of glycerol; biochemical use and physiological action of glycerine; and the most important industrial and medicinal uses.

From the summary of contents above, it is obvious that the main forte of this book is undoubtedly its completeness. The abundant use of tabular form in presenting data also makes the book convenient to use and easy to read. It is not designed to be an extremely technical book, but only to serve as a complete reference to the modern technology of glycerol. The book most certainly upholds the reputation of excellence maintained by the many members of the ACS Monograph series. The editors also accomplish their avowed purpose of providing the connecting link between the classical work and the most recent advances in the field. The book is written, however, so as to contain something valuable for everyone associated with the field from the research chemist to the purchasing agent.

D. H. RUSSELL

RADIO AND RADAR TECHNIQUE, by A. T. Starr. 812 pages, diagrams, 16×24 cm. New York, Pitman Publishing Corp., 1953. Price, $15.00.

A reviewer must keep well in mind his obligations to the authors, to the publishers and to possible readers of the books he reviews. One of these obligations is that his review should be prompt, for timeliness is of the essence of literature so transitory as book reviews. Another obligation is that he should read the books in reasonable detail. Surely he must expect to spend a perceptible fraction of the time the author gave in the preparation of the book.

Sometimes a book appears which sharply emphasizes the dilemma these two obligations can pose. Dr. Starr's book is one such; its seven chapters cover 510 pages of closely packed text. These are followed by 30 appendices in 281 more pages. This reviewer confesses that he has done little more than skim through the book, with such variation in detailed study as his personal interests and experience dictated.

The seven chapters cover methods of communication and their limitations, electromagnetic theory, microwave and short-wave techniques, antennas, vacuum tubes, steady-state circuit techniques, and transient circuit techniques. The appendices cover special mathematical methods, the formulation of physical theories with such mathematical tools, engineering applications and tabular material as transform pairs and the International Teleprinter code.

The author's aim, as stated in his preface, is to cover the essentials of the radio and electronic field in one volume. To reduce the bulk, he has treated methods and techniques only, avoiding discussion of systems and their applications. In the reviewer's opinion the objective is praiseworthy, but likely impossible of accomplishment. The number of subjects which may be considered as essential is subject to some variation with personal opinion. Nevertheless, it is large. Dr. Starr has ninety-six major sub-headings in his seven chapters. Necessarily the treatment of each must be brief, and often the result is so compact as to be readable only by a person so familiar with the subject that he does not need the book. It is felt that some expansion of important material could have been attained if other material, such as an

introduction to vector anaylsis, the elements of network analysis, and Fourier methods, for example, had been omitted, under the assumption that the reader's basic education had covered these.

There is an extensive bibliography, which is characterized by a number of typographic errors in author's names, suggesting poor proofreading, and by several curious omissions. The book draws heavily on the MIT Radiation Laboratory Series, to individual volumes of which reference is frequently made. But Silver's *Microwave Antenna Theory and Design*, and Lawson and Uhlenbeck's *Threshold Signals* are nowhere mentioned.

A. C. BYERS

NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS, by William Edmund Milne. 275 pages, 15×23 cm. New York, John Wiley & Sons, Inc., 1953. Price, $6.50.

The author introduces the subject in a brief chapter on the need for and origin of numerical methods of solving differential equations. He suggests an examination of the differential equation including the plotting of a few isoclinical curves before undertaking a numerical solution. Chapter two is used to familiarize the reader with "quick and dirty" methods that are important as a basis for a good understanding of the more refined methods introduced later. The methods treated include: Method I, Point-slope formula; Method II, Trapezoidal formula; Method III, Analytic continuation; and Method IV, Successive substitutions. In a chapter entitled "Methods Based on Numerical Integration" three additional ones are discussed: Method V, Forward integration; Method VI, Successive Integrations; and Method VII, Forward and successive integrations. Chapter 5 is devoted to methods of Runge-Kutta and methods on higher derivatives. It includes: Method VIII, Kutta's fourth-order method; and Method IX, Formulas with first and second derivatives. Variations of the numbered methods are treated throughout, and the various methods are compared.

The methods mentioned so far were developed for and illustrated by a single differential equation of the first order. Chapter 6 extends these to systems of first-order equations and to equations of higher order. Four more methods are introduced. An additional chapter on "Two-Point Boundary Conditions" concludes Part I on Ordinary Equations.

The second part treats partial equations. It consists of four chapters: Explicit Methods. Parabolic and Hyperbolic Equations; Linear Equations and Matrices; Implicit Methods. Elliptic Equations; and Characteristic Numbers.

Very brief Appendices on "Round-Off Errors," "Large-Scale Computing Machines," and "The Monte Carlo Method" serve merely to introduce these subjects and refer the reader to a wealth of literature in the bibliography. This includes 258 references. Though the question of round-off errors is not adequately treated, analysis of errors sections discuss the errors of the methods presented.

This book is timely due to the large amount of work now being done on large-scale, general-purpose digital computing machines. Ample examples make it ideal for a text, and anyone working in numerical methods will want to add it to his bookshelf. A book on this subject was badly needed and the Wiley Applied Mathematics Series is fortunate to have Milne's text admirably fill this need.

DONALD B. HOUGHTON

*Quantum Chemistry*, by Kenneth S. Pitzer, 525 pages, diagrams, 15×22 cm. New York, Prentice-Hall, Inc., 1953. Price, $7.50.

There have existed for a number of years several well-known textbooks of quantum mechanics which were written mainly for chemists. Chief among these are *Introduction to Quantum Mechanics* by Pauling and Wilson and *Quantum Chemistry* by Eyring, Walter, and Kimball. These however, still left much to be desired. Fortunately it can be reported that the gap is now being well filled. In 1952 there was published a significant volume entitled *Valence* by C. A. Coulson, and now Professor Pitzer's book rounds out the list very nicely.

*Quantum Chemistry* first appeared under the title "Quantum Theory, with Applications to Chemistry" as a set of mimeographed notes for Professor Pitzer's course at the University of California at Berkeley. According to the author, it was designed to meet the needs of the "typical physical chemist."

The first six chapters in the book constitute a survey of the principles of quantum mechanics. After that there are seven chapters of applications to chemistry. These seven chapters bear the following titles: Kinetic Theory and Statistics, Chemical Bonds and Valence, Molecular Spectra and Thermodynamic Properties of Perfect Gases, Crystalline Solids, Imperfect Gases and Liquids, Miscellaneous Topics, and Nuclear Phenomena. There are twenty-four appendices in which details are given which were left out of the main body of text. Typical examples of the material which is deferred to the appendices are Details of the Heitler-London Treatment of $H_2$, Symmetry and Normal Vibrations, and Second Order Perturbation Theory. At the end of each chapter the author gives a list of general references which the student will no doubt find very useful. There are also some excellent questions and problems at the end of most of the chapters.

To one who has a good foundation in the various subjects which are linked with quantum chemistry the best feature of the book is likely to be the considerable extent of the subject matter. For that student, the text will probably tie together many studies which previously seemed only slightly related, and it will give more meaning to the whole of physical chemistry. To one less well prepared, however, the book may seem deficient in explanations of fundamental points. This may not be a valid criticism, however, because no book can cover an extensive and fundamental subject to the satisfaction of everyone. It is the opinion of this reviewer that one can not learn quantum theory merely by reading a book, for the subject involves too many concepts which are alien to everyday experience. The student must study from not one, but from several texts; he must discuss the subject with others and argue points which disturb him; he must live with this new set of ideas before he can claim to know the subject; finally, he must commit himself to continual review. Professor Pitzer's new book will serve very well in all phases of such a program of learning and review: it will be a good text for the student who is just making the acquaintance of the quantum theory, and it will serve admirably as a review for the professional physical chemist. PHILIP L. HANST

SYNCHROS, SELF-SYNCHRONOUS DEVICES AND ELECTRICAL SERVO-MECHANISMS, by Leonard R. Crow. 222 pages, diagrams and photographs, 14×22 cm. Vincennes (Indiana), Scientific Book Publishing Co., 1953. Price, $4.20.

The synchro, common name for a number of commercial devices which are used to transmit shaft rotational information by wire, is one of the frequently used components in modern electrical instrumentation. In its various forms the synchro is not complex as to application; however, its design involves magnetic circuit and field problems associated with electric motor development. Primarily, Leonard Crow's book covers considerations of synchro application and the qualitative theory of operation within the device. The complex electromagnetic problems which face the designer of the synchro are not treated in this book, and it is not intended as that sort of reference.

However, much useful information regarding synchro circuits can be derived from this well illustrated and vividly written monograph. It contains excellent diagrams of rotating machines and machine winding systems. It also contains photographs of interesting models of elementary synchros creditable to the author's extensive teaching experience which included the development of training aids for teaching electricity.

Mathematics of the basic trigonometric voltage and current relations is not carried beyond the high school level and is easily understood by the average technician. Reference is made to contemporary commercial units and practical installations in which d-c. self-synchronous devices as well as a-c. synchros are described. The author touches on the subject of servo-mechanisms in an elementary fashion. This is done by wrapping up such subjects as feed-back and transient response and placing them in a box labeled "amplifier and anti-hunt circuits."

A serious shortcoming of the book is the lack of an index. For instance, the word "amplidine" is not mentioned in the table of contents, and one finds only after thumbing through the text that a discussion of this important device as well as an illustration has been included. Occasionally the discussion becomes somewhat wordy, but, taken as a whole, the volume is useful in explaining the

rudiments of synchro and servo devices including ways in which they may be connected to achieve various results.

C. W. HARGENS

THE PHILOSOPHY OF SCIENCE, by Stephen Toulmin. 176 pages, 13×19 cm. New York, Longmans, Green and Co., Inc.; London, Hutchinson's University Library; 1953. Price: $1.80 (text); $2.25 (trade).

This volume is an addition to the "Home University Library" which has gained a well-established reputation in Britain and deserves to be better known in this country. This present volume is in keeping with the high standard that has been set and maintained in the philosophy section of the series. It is not a profound work, being addressed to those who have no special knowledge of either mathematics or natural science, but it will appeal to the student who is not afraid to do some serious thinking instead of having it done for him.

With the aid of elementary examples the author shows the non-technical reader some of the more important things he needs to know about the logic of the physical sciences. The author realizes the difficulties which beset such readers when confronted with self-contradictory terms, such as "invisible light," and perplexities like "the curvature of space." His aim is to furnish his readers with the knowledge which will enable them to penetrate the screen of technical terms in order to reach the living body of meaning. It is regarded as vital to the understanding of any account of a theory that the inquirer should know what questions to ask. To indicate what are these questions is the principal aim of the book, and the author is more concerned with the sort of things the scientists do with the words they employ than he is with the things they have to say.

Being only designed as an introduction to the subject, the book does not deal with a wide field. A chapter on discovery is a painstaking effort to describe what is meant when something is "discovered" in physics, and how it differs from other forms of discovery. A second chapter seeks to determine what a scientist means in speaking of a law of nature, and points out how a law differs from a principle. The chapter on "Theories and Maps" draws an analogy between the two to show how a limited number of highly precise and well-chosen measurements and observations can be employed to produce an outline from which can be read an unlimited number of facts of almost as great a precision. A final chapter on "Uniformity and Determinism" explains (not too clearly, by the way) what is meant when the scientist speaks, as he does with increasing frequency, of determinism.

The serious reader should gain a clear comprehension of what such important expressions as "theories," "hypotheses," "discoveries," and "laws of nature" mean when used by the scientist, and he will learn that these terms have a precise meaning that is not associated with them in everyday speech. The book will serve as a guide to the man who wants to know why the terms have different meanings to the writer on popular science and to the professional scientist.

T. C.

KINEMATICS OF MECHANISMS, by N. Rosenauer and A. H. Willis. 395 pages, diagrams, 15×22 cm. Sydney (Australia), Associated General Publications, Pty. Ltd., 1953. Price, £ (Aust.) 4/10/0.

Both authors studied and taught kinematics of mechanisms in Europe before being called to the New South Wales University of Technology, Australia. This subject has received greater attention and consequently it has developed further in Europe than in America. The main contribution of this book is the presentation by graphical methods of the kinematics now being taught in Europe using mathematical analysis. The graphical approach is extended to Coriolis acceleration, which in other texts requires a slide rule calculation in the midst of a graphical analysis. This method aids the analyst in visualizing the action of the mechanism.

A consistent and convenient system of notation has been used throughout the text. No titles have been used beneath figures. This means that the text must be read carefully for comprehension. It, therefore, loses some of its value as a reference book. Many examples are worked out in the text, but very few are suggested as exercises.

The concluding chapter is entitled, "An Introduction to the Synthesis of Mechanisms." It is an attempt to design by ra-

tional methods, mechanisms which are required to perform particular functions. Here also graphical methods are found to be most convenient and powerful.

The subject is clearly developed in this text. Terms are carefully designated and well introduced. The illustrations are not as carefully drawn as those prepared by publishers of technical material, but they well illustrate the principles. This text is recommended for analysts desiring to use graphical solutions. A. DONALD HAY

QUALITATIVE ANALYSIS AND ANALYTICAL CHEMICAL SEPARATIONS, by P. W. West, M. M. Vick, A. L. LeRosen. 223 pages, diagrams, 15×22 cm. New York, The Macmillan Co., 1953. Price, $3.75.

This book contains an elementary exposition of qualitative chemical analysis, together with a brief survey of basic principles, which are applied in qualitative analytical chemistry. The purpose of the book is to present a new analytical procedure based on the early separation of tri- and tetravalent ions. The sequence suggested is the following: Group 1 ($Na^+$, $K^+$, $NH_4^+$); Group 2 ($Ag^+$, $Hg_2^{++}$); Group 3 ($Sn^{+4}$, $Bi^{+3}$, $Sb^{+3}$, $Fe^{+3}$, $Al^{+3}$, $Cr^{+3}$); Group 4 ($Pb^{+2}$, $Ba^{+2}$, $Sr^{+2}$, $Ca^{+2}$); Group 5 ($Mn^{+2}$, $Hg^{+2}$, $Fe^{+2}$, $Cu^{+2}$, $Co^{+2}$, $Ni^{+2}$, $Cd^{+2}$); Group 6 ($Sn^{+2}$, $As^{+3}$, $Zn^{+2}$). A few pages are devoted to anions separation. Problems and questions are provided at the end of each chapter.

The book is mainly written as a classroom guide and should prove useful for that purpose. G. PARRAVANO

CHEMICAL ANALYSIS OF CAST IRON AND FOUNDRY MATERIALS, by W. Westwood and A. Mayer. 565 pages, diagrams, 16×24 cm. New York, Pitman Publishing Corp.; London, George Allen and Unwin Ltd.; 1951. Price, $8.50.

The art of founding of the cast metals has in recent years been greatly modified by the impact of science. A considerable literature has grown up and much unrecorded experience has been gained. The authors have taken advantage of the opportunities afforded them through experience with the British Cast Iron Research Association and collaboration with the Methods of Analysis Committee of the British Iron and Steel Research Association to offer to the industry an authoritative modern work.

Many important developments have recently occurred in the sphere of metallurgical analysis which have here been given consideration, namely the mercury cathode separation, the Spekker photoelectric absorptiometer, electrolytic titrations, the constantly increasing use of organic reagents and the pH indicators which are now readily obtainable.

The book is divided into nine main sections: Introduction, Laboratory Practice and Apparatus; Sampling and Analysis of Cast Iron; Sampling and Analysis of Ferro-Alloys, Inoculants and Ladle Additions; Sampling and Analysis of Iron Ore and Used Annealing Ore; Analysis of Cupola, Rotary Furnace and Blast Furnace Slags; Sampling and Analysis of Sand, Siliceous Refractories and Firebricks; Sampling and Analysis of Coke; Analysis of Coal Dust; and Sampling and Analysis of Linseed Oil.

Section One consists of the theoretical considerations to give laboratory apprentices and assistants a necessary background. The authors consider that this background is essential to the practice of analytical chemistry. Chemical equations are given in the molecular form rather than ionic for simplicity of understanding. This section deals with reagents, solution, fusion, general separations (mercury cathode, ammonia, bicarbonate, sodium hydroxide, zinc oxide and hydrogen sulfide), precipitation, filtration (paper pulp pad, asbestos pad, sintered glass and porous alumina), washing of precipitates, drying and ignition of precipitates, weighing, glassware, platinum apparatus and the Spikker photoelectric absorptiometer.

Section Two deals with the very difficult and controversial subject of sampling, and it is very simply explained. This section also deals with the analysis of cast iron constituents or contaminants. The elements are arranged in alphabetical order, and in most cases several procedures have been described in detail. The division for each element consists of the principal of the method; reagents (the percentages and dilutions are all noted); procedure; calculation; theoretical considerations; time, range and reproducibility of results and an extensive bibliography.

Section Three considers the sampling and analysis of specific alloys. Analysis for each major constituent is noted along with certain consistent contaminants. The analysis for certain of these contaminants refers the reader back to Section Two; however there are additional suggestions if the analysis deviates from the standard procedure in any way.

The successive sections are self-explanatory from their headings.

The book is concluded with twelve appendices which are very pertinent to the materials included. There are a number of very fine illustrations of apparatus and techniques which are distinctly described.

Although many methods contained in this book are primarily designed for the analysis of foundry materials, the experienced analyst will find much of application to general analysis. PHYLLIS A. PARKIN

## BOOK NOTES

CHEMICAL PROCESS MACHINERY, by E. Raymond Riegel. Second edition, 735 pages, diagrams, illustrations, 16×24 cm. New York, Reinhold Publishing Corp., 1953. Price, $12.50.

Since 1944, when the first edition of this widely used reference work was published, many new items have become commercially available, while others are now obsolete. This present revision brings the material up-to-date, with new illustrations and tables to explain recent developments. The classification of equipment is still by function, as in the first edition. New equipment described includes multispheres, high-speed centrifugal separators, the molecular still, the turbo-drier, and many others. In addition to new machinery, the text includes new descriptive material on many of the standard chemical processes. The book should prove invaluable to those in chemical industries.

DESIGN OF MACHINE ELEMENTS, by M. F. Spotts. Second edition, 504 pages, diagrams, 16×24 cm. New York, Prentice-Hall, Inc., 1953. Price, $7.25.

In this enlarged second edition of a proven text for undergraduate courses in machine design, the author has retained the academic approach, but has enhanced its usefulness by the inclusion of a great many practical problems. Several sections have been rewritten—notably those on gears, cams and working stresses—to include more detailed discussion. New material embraces design data for partial journal bearings, for standard-size spring wire and rubber springs, and for shafting.

FUNDAMENTALS OF PHYSICAL SCIENCE, by Konrad Bates Krauskopf. Third edition, 694 pages, illustrations, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $6.00.

Although this text was revised in 1948, science has progressed so much in the past five years that a new edition has been prepared. New material has been added on electronics, on silicones, and on recent theories about the origin of the planets. Major revisions have been made in the discussions of the atomic nucleus, atomic energy, cosmic rays; and other subjects have undergone minor revision. Many new illustrations have been added, and less essential material has been set in smaller type. A list of visual aids, with a short description of each, should prove helpful to instructors who can take advantage of it.

INTRODUCTION TO ENGINEERING ECONOMY, by Baldwin M. Woods and E. Paul DeGarmo. Second edition, 519 pages, diagrams, 15×22 cm. New York, The Macmillan Company, 1953. Price, $6.00.

In this second edition, the authors have revised the original text to increase clarity and to incorporate the suggestions resulting from eleven years of teaching from it. Of interest among the relatively few new topics is a section on effects of income tax. Among the original subjects extensively revised is the chapter on government engineering projects and the effect on the economy. Although intended as a text for college students, the book will prove of interest to practicing engineers and to others interested in the changing economics of industry.

ORGANIC CHEMSITRY, by Ray Q. Brewster. Second edition, 855 pages, diigrams, 16×24 cm. New York, Prentice-Hall, Inc., 1953. Price, $7.00.

Dr. Brewster, in revising his text, has enlarged parts of it and still kept the size approximately the same. This has been accomplished in part by the use of smaller type for some of the formulas and in part by omitting some of the descriptive material on dyes, sterols, terpenes and alkaloids. The new material has been added to the sections on acetylene, fluorine compounds, the Walden inversion, free radical mechanisms, etc. Reorganization of some sections has grouped aliphatic nitro compounds, amines and cyanides into one chapter; one short new chapter on organo-metallic compounds has been added. Also, new charts and new problems serve to enhance this new edition.

TOOL ENGINEERS' DATA BOOK, by Gerhard J. Gruen. 219 pages, tables, 16×24 cm. New York, Reinhold Publishing Corp., 1953. Price $5.50.

Tool engineers and designers will welcome this small reference book, in which are found all the tables, formulas, constants and specifications necessary to their profession. No theoretical discussions and no descriptive passages are given; instead, only facts pertinent to tool design and use are set forth in their most compact form. The author has purposely omitted the textual material usually found in engineering handbooks, believing that practicing engineers do not have the time to search through massive volumes. His aim—to help the engineer obtain results quickly and accurately—is a laudable one and merits the thanks of busy tool engineers.

CALCULUS, by C. R. Wylie, Jr. 565 pages, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $6.00.

This reviewer, whose college calculus courses were undertaken some twenty years ago, when it was not considered necessary to "make the subject interesting," was glad to examine this 1953 version of a calculus text. While the basic principles have not changed in twenty years, some new features are immediately evident. These include: the informal tone of the text material—"we" being much in evidence; the use of boldface type within the text to indicate definition of terms (which is presumably a device to save the student both the effort of using the index and of deciding for himself what statements are important); and the use of problems which have actual application instead of being merely exercises in the use of the calculus. This last feature seems to be the one which will prove most helpful in the classroom.

COLLEGE CHEMISTRY—A SYSTEMATIC APPROACH, by Harry H. Sisler, Calvin A. VanderWerf and Arthur W. Davidson. 623 pages, illustrations, 16×24 cm. New York, The Macmillan Company, 1953. Price, $5.25.

The authors of *General Chemistry—A Systematic Approach* now offer essentially the same text in a much shorter, more compact form. The same logical treatment is used in the shorter version, which has another advantage in that it is up-to-date, at least to January 1953, in regard to practical and theoretical developments. Emphasis is again on the "thinking-through" approach to chemistry rather than on the "rote memory" one. The text should be easy to teach and should provide a sound first course for chemistry students.

RINEHART MATHEMATICAL TABLES, FORMULAS AND CURVES, compiled by Harold D. Larsen. Enlarged edition, 280 pages, tables, 14×22 cm. New York, Rinehart & Company, Inc., 1953. Price, $2.00.

There is no need to "review" a second edition of this excellent handbook for mathematicians, engineers, physicists and statisticians, other than to note that seven tables have been added in the field of applied mathematics. These new tables cover: Values and Logarithms of the Gamma Function; The Probablility Integral $\frac{2}{\pi}\int_0^t e^{-t^2}dt$; Values of Bessel Functions $J_0(x)$ and $J_1(x)$; Values of the Complete Elliptic Integrals; Values of the Elliptic Integral of the First Kind: $F(k,\theta)$; Values of the Elliptic Integral of the Second Kind: $E(k,\theta)$; and Square Root Divisors. Thorough checking of proofs and of source material leads the author to claim these tables as the most accurate ones available.

## CURRENT TOPICS

**Industrial "Erector Set."**—Substantial savings in time and money are being achieved through the use of an industrial "Erector set" by the General Electric Company's J-47 jet engine plant at Evendale, according to L. E. Good, Manager of Project Manufacturing.

The plant, which is headquarters for the company's Aircraft Gas Turbine Division, is the largest producer of J-47 jet engines for Air Force and Navy fighters and bombers.

The Wharton system consists of a set of approximately 450 basic pieces which are used to assemble jigs and fixtures for machining and assembly work. These jigs and fixtures are used in drilling, reaming, tapping, milling, and many other metal working processes in the manufacture of jet engine parts.

The set is composed of interlocking blocks, clamps, bolts, bushings, and other parts which can be assembled in an almost infinite number of combinations.

"Substantial time savings in manufacturing parts in small quantities is achieved by the new system," Mr. Good said.

"By using the 'Erector set' principle, which is a British invention called the Wharton Universal jig and fixture system, a considerable saving can be made by having toolmakers quickly assemble dependable 'temporary' tooling to almost any design," Mr. Good explained.

"When the job is completed, the tooling can be disassembled for use on another part. As a result, money is not invested in permanent type tooling for short runs," the engineer said. Formerly, most design changes would have involved making complete new fixtures which are both costly and time consuming.

**Lysine Polypeptides Synthesized.**—University of Wisconsin scientists have synthesized substances known as lysine polypeptides and related compounds which affect some viruses and bacteria in much the same way as the antibodies that create immunity.

The finding that these polypeptides would act in much the same manner upon viruses and bacteria as immune serum has been reported by Prof. Mark A. Stahmann, Maurice Green, and associates in the UW department of biochemistry.

While the polypeptides themselves are at present probably too toxic to be used against disease germs in humans, it seems possible that a field of investigation is opening which eventually may give medicine new weapons against bacterial and virus infections.

The scientists found that the lysine polypeptides forced virus particles and bacteria to clump together. Growth and respiration were inhibited. This agglutination, as it is known, is exactly what happens to bacteria and viruses when they are attacked by natural body defense agents.

The similarity between the action of natural antibodies and synthetic polypeptides is sufficient to warrant some hope that eventually chemical structures which may in part duplicate the action of antibodies can be synthesized in the laboratory, Prof. Stahmann says.

Among the micro-organisms against which the scientists found the lysine

polypeptides to be effective are the influenza virus, fowl bronchitis virus, Newcastle disease virus, tobacco mosaic virus, a bacteriophage or virus attacking bacteria, the bacillus that causes anthrax, and a number of other common and infectious bacteria species.

Prof. Stahmann and his associates have put forth the theory that the polypeptides combine with acidic surface molecules of virus particles or bacteria with which they come in contact, thus effectively closing the door to needed food and oxygen.

Again, this is like the thing that happens when the living animal body takes up arms against an invasion of disease germs, Prof. Stahmann points out. During such an invasion, the body produces antibodies which congregate on the surface of the disease organisms, rendering them powerless to obtain food and oxygen. The antibodies also cause the disease organisms to clump, which helps render them easy prey to defending white blood cells.

**"Flying Watchman."**—An electronic fire detector for aircraft that gives continuous protection has recently been introduced. The device's sensing element, which may be as long as 100 ft., can be wound like a spider web throughout areas where fires are likely to break out. The detector has no moving parts and automatically resets itself after signaling the presence of a fire. The tube and wire of the sensing element are made of Inconel, a nickel-chromium alloy, so they can withstand temperatures up to 2000 F. without damage or replacement..

**New Light for Stereo Photography.**—Mayfair Munufacturing Company of Brooklyn has developed a new light for Stereo photography. Now, for the first time, owners of Stereo cameras can take indoor pictures with a lightweight, portable unit using ordinary inexpensive photo flood bulbs. Mayfair Stereolite's construction gives controlled balanced illumination resulting in true-to-life stereo pictures. Of special interest to Stereo fans who also own movie cameras is the rotating camera platform, which has two independent camera mounting stations—one for Stereo, the other for movies. This double purpose feature saves money because Stereolite can also be used for shooting movies—without the addition of accessories. Another important feature—movable arms—permits operator to concentrate light and control shadows.

Electrically, Mayfair's Stereolite is provided with a dimmer switch, offering three levels of light from dim to bright.

**A Cushioning Material for Radar Darkrooms.**—A new absorbent material has been developed by the Navy for use in creating "radar darkrooms." Created by chemists in the Naval Research Laboratory of the Office of Naval Research, the material facilitates study of extremely high frequency antenna patterns (2500 to 35,000 megacycles), since it effectively cushions bounce-back from walls and ceiling. Called "Darkflex," the material uses rubber-bonded curled animal hair, which is inexpensive, light in weight, convenient to handle and readily available.

**"Flying" Boats Without Wings.**—Several hydrofoil boats of different types are now being evaluated by the Navy. Hydrofoils are "underwater wings" installed on the bottom of a small craft which allow it to "fly" in the water as an aircraft flies in the air. When "flying" speed is reached, the hull lifts from the water so that the

boat is entirely supported by its underwater wings. The hydrofoil, similar in design to an airfoil, produces lift in essentially the same manner as an aircraft wing. Although the speed of the Navy experimental boats is classified, there is a Swedish hydrofoil ferryboat which has a speed of about 35 knots; during World War II German scientists developed a hydrofoil boat having a speed of about 50 knots.

**Australian Midget Radio.**—A new midget radio receiver-transmitter has been developed for the Australian Army and is being put into service after a year's testing. It is an extremely reliable set and is easily operated and maintained; it is half the weight of the sets now in use and is a superior performer; it can be carried in two ammunition pouches on a man's belt or can be used as a ground station. According to the Minister for Supply, "No set so light, compact and powerful has been developed anywhere else in the free world."

**Ultrasonic Machine Tool.**—A new kind of machine tool that carves fancy shapes, drills holes, etc. in hard substances, is driven by ultrasonic sound wave which is pitched so high that human ears cannot hear it, according to an announcement by the Raytheon Manufacturing Co. The motion of the tool is invisible, for it travels only a few thousandths of an inch each way. Yet the extreme speed of this motion is the key to its efficiency. One of its most amazing features is that the cutting tool can be made of relatively soft material, yet cut its way through hard steel, stone, cast iron, or even sapphire.

**The New Uses for Western Union.**—Weather forecasts by wire were offered, beginning this spring, by the National Weather Institute of Los Angeles. The service covers both short-range weather predictions and long-range ones for periods of from one to six months in advance. A subscriber may request information for a particular geographical area or for particular dates, and farmers are expected to be the principal users.

Railroad reservations were made recently at the Pittsburgh office of the Pennsylvania Railroad by means of the Western Union Telegraph Company's Ultrafax system, for the first time in railroad history. An entirely new system of selling railroad tickets and making space reservations was inaugurated when duplicates of space coupons were handed to customers while the originals were still at the main office. Fred E. Moran, Superintendent of the Western Union at Pittsburgh, said that the use of Intrafax has provided the railroads with a new means of eliminating communications bottlenecks, as well as of solving the problem of speeding internal correspondence of large companies.

**Varistors.**—A new line of Varistors (non-linear resistors) has been introduced by the International Resistance Company's Special Products Division in the Philadelphia plant.

These units have many applications in circuits where sharp variation of resistance with applied voltage is required, and are available in five convenient cell sizes in a wide variety of enclosures. Designed to conform with MIL and JAN Specifications on humidity, shock, vibration, temperature cycling, solder pot, fungus resistance, etc., IRC Varistors have unusually low shunt capacitance and can be used effectively in RF circuits —the response is instantaneous.

**Rubber Shipping Containers.**—A new type of shipping container has been tested recently at the U. S. Naval Engineering Experiment Station, Annapolis, Md. It's a drum made of synthetic rubber, originally developed to be a competitor for metal or glass containers used in shipping corrosive liquid chemicals. But already it's been found adaptable to several unexpected uses. Samples: shipment of wheat flour from mill to baker; parachute drops of water for fighting forest fires. Tests so far indicate the drums are also suitable for transporting petroleum products, anti-freeze solutions, cleaning solutions, paints, and selected liquid food products. Principal advantages: (1) light weight, (2) collapsibility when empty, (3) absence of rust and corrosion, (4) resilience and (5) substitution for strategic steel. Disadvantages: (1) high initial cost, (2) temporarily poor procurement potential and (3) non-adaptability to existing cleaning and loading machines. Currently the containers are manufactured in 55, 600 and 3000 gallon sizes.

**New Particle-Size Distribution Analyzer.**—Sharples Corporation Research Laboratories, Bridgeport, Pa., have developed a new particle-size distribution analyzer, known as the Sharples Micromerograph. Now in production, this high precision instrument should be of great interest to all industries producing or handling powders of any kind, since it offers a quick and accurate means of analyzing the particle-size distribution in such powders.

Some idea of its speed and accuracy may be realized by the fact that it takes three technicians about four months to count 70,000 particles of a certain powder, with a probable error of $\pm 150\%$, while the Micromerograph counts billions of particles in an hour, with a probable error of $\pm 3\%$.

In operation, the instrument uses a 100-mg. sample of powder, which is blown by a high pressure nitrogen charge through a deagglomerator slit, where the powder is dispersed by the viscous shearing forces of the gas. The particles then enter a sedimentation chamber—a thermally insulated aluminum tube about 4 in. in diameter and about 7 ft. long—and fall, through the action of gravity, onto a weighing pan at the bottom of this aluminum tube. The weighing system is an automatic, null seeking, servo-mechanical recording balance. The recording indicates the cumulative weight collected as a function of time. By use of Stokes' Law of Fall, the time of fall can be converted into a value for $d\sqrt{\rho}$, where $d$ is the diameter of a sphere which falls at the same rate and $\rho$ is the true density of the material.

In addition to providing continuous data and drastically reducing the time required for a test run, the Micromerograph offers other advantages, such as wide size range, attention free operation, simple sample preparation and positive deagglomeration of the powder. Skilled technicians are not required for its operation.

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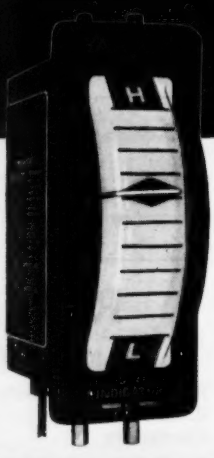


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*(Revised to May, 1953.)*

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